TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

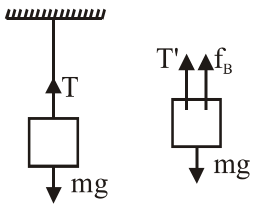
PHYSICS

1. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is :

(1) 4.0mm (2) 3.0mm (3) 5.0mm (4) zero

Ans (2)

Sol.



$$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$$

$\Delta\ell \propto F$ .....(i)

$T = mg$

$$T = mg - f_B = mg - \frac{m}{\rho_b}.\rho_\ell.g$$

$$= \left(1 - \frac{\rho_\ell}{\rho_b}\right) mg$$

$$= \left(1 - \frac{2}{8}\right) mg$$

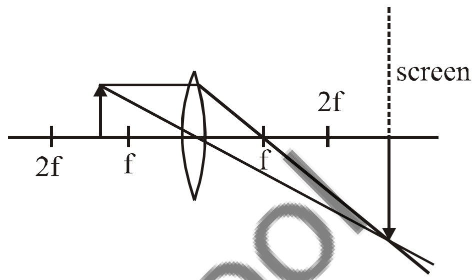
$$T' = \frac{3}{4} mg$$

From (i)

$$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$$

$$\Delta\ell' = \frac{3}{4} \cdot \Delta\ell = 3 \text{ mm}$$

2. Formation of real image using a biconvex lens is shown below :



If the whole set up is immersed in water without disturbing the object and the screen position, what will one observe on the screen?

(1) Image disappears
(2) No change
(3) Erect real image
(4) Magnified image

Ans (1)

Sol. From $\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Focal length of lens will change hence image disappears from the screen.

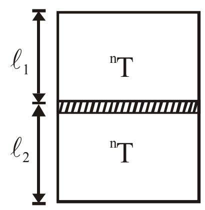
3. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is ℓ_1, and that below the piston is ℓ_2 , such that $\ell_1 > \ell_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by :
(R is universal gas constant and g is the acceleration due to gravity)

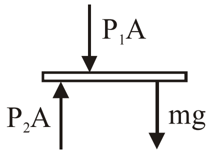
(1) $\frac{nRT}{g}\left[\frac{1}{\ell_2} + \frac{1}{\ell_1}\right]$ (2) $\frac{nRT}{g}\left[\frac{\ell_1 - \ell_2}{\ell_1\ell_2}\right]$

(3) $\frac{RT}{g}\left[\frac{2\ell_1 + \ell_2}{\ell_1\ell_2}\right]$ (4) $\frac{RT}{ng}\left[\frac{\ell_1 - 3\ell_2}{\ell_1\ell_2}\right]$

Ans (2)

Sol.





$P_2A = P_1A + mg$

$$\frac{nRT.A}{A\ell_2} = \frac{nRT.A}{A\ell_1} + mg$$

$$nRT\left(\frac{1}{\ell_2} - \frac{1}{\ell_1}\right) = mg$$

$$m = \frac{nRT}{g}\left(\frac{\ell_1 - \ell_2}{\ell_1.\ell_2}\right)$$

4. A simple harmonic motion is represented by:

$y = 5(\sin 3\pi t + \sqrt{3}\ \cos 3\pi t)$ cm

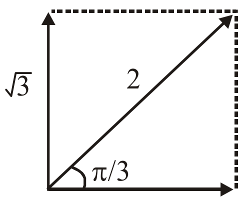
The amplitude and time period of the motion are:

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s

(3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

Ans. (4)

Sol.



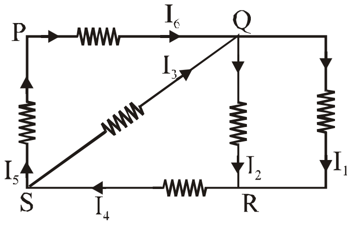
$$y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$$

$$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Amplitude = 10 cm

$$T = \frac{2\pi}{w} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$$

5. In the given circuit diagram, the currents, $I_1 = -0.3$A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2,I_3 and I_6,respectively, are :



(1) 1.1 A, 0.4 A, 0.4 A

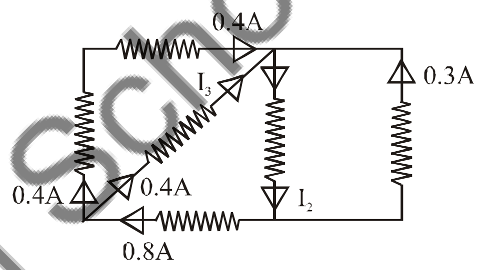
(2) –0.4 A, 0.4 A, 1.1 A

(3) 0.4 A, 1.1 A, 0.4 A

(4) 1.1 A,–0.4 A, 0.4 A

Ans. (1)

Sol.



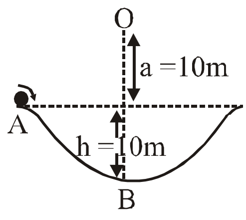
From KCL, $I_3 = 0.8 - 0.4 = 0.4$A

$I_2 = 0.4 + 0.4 + 0.3$

$= 1.1$ A

$I_6 = 0.4$A

6. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g= 10 m/s²)



(1) 8kg-m²/s (2) 6kg-m²/s

(3) 3kg-m²/s (4) 2kg-m²/s

Ans. (2)

Sol. Work Energy Theorem from A to B

$$mgh = \frac{1}{g}m\nu_B^2 - \frac{1}{g}m\nu_A^2$$

$$2gh = \nu_B^2 - \nu_A^2$$

$$2\times10\times10 = \nu_B^2 - 5^2$$

$$\nu_B = 15\,m/s$$

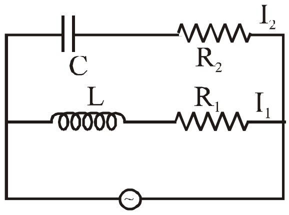
Angular momentum about 0

$L_0 = m\nu r$

$= 20\times10^{-3}\times20$

$L_0 = 6$ kg.m^2/s

7.



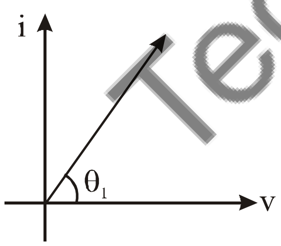
In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by

V= $200\sqrt{2}$ sin(100t) volts. The phase difference between I_1 and I_2 is :

(1) 30° (2) 0° (3) 90° (4) 60°

Ans. **(Bonus)**

Sol.



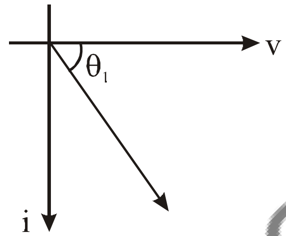
$$x_e = \frac{1}{\omega_c} = \frac{4}{10^{-6}\times\sqrt{3}\times100} = \frac{2\times10^4}{\sqrt{3}}$$

$$\tan\theta/2\ \frac{x_e}{R_e} = \frac{10^3}{\sqrt{3}}$$

θ_1 is close to 90

For L–R circuit

$$x_L = w_L = 100\times\frac{\sqrt{3}}{10} = \sqrt{3}$$



$R_1 = 10$

$$\tan\theta_2 = \frac{x_e}{R}$$

$\tan\theta_2 = \sqrt{3}$

$\theta_2 = 60°$

So phase difference comes out 90 + 60 = 150.

Therefore Ans. is Bonus

If R_2 is 20 KΩ

then phase difference comes out to be 60+30 = 90°

8. A paramagnetic material has 10^{28}atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8 $\times10^{-4}$. Its susceptibility at 300 K is :

(1) 3.672×10^{-4} (2) 3.726×10^{-4}

(3) 3.267×10^{-4} (4) 2.672×10^{-4}

Ans (3)

Sol. $x \ \alpha\ \frac{1}{T_C}$

curie law for paramagnetic substane

$$\frac{x_1}{x_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8\times10^{-4}}{x_2} = \frac{300}{350}$$

$$x_2 = \frac{2.8\times350\times10^{-4}}{300}$$

$= 3.266\times10^{-4}$

9. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0 ms^{-1}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3 \times 10^{-4} Wb/m^2$. The value of the induced emf in wire is :

(1) $2.5 \times 10^{-3} V$ (2) $1.1 \times 10^{-3} V$

(3) $0.3 \times 10^{-3} V$ (4) $1.5 \times 10^{-3} V$

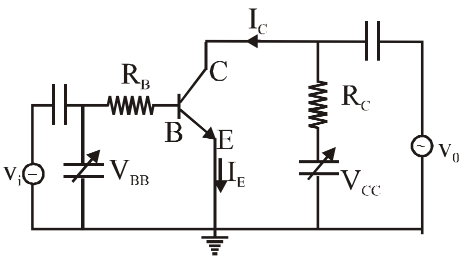
Ans (4)

Sol. Induced emf = $Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 10$

$= 1.5 \times 10^{-3}$ V

10.



In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5V$, $\beta_{dc} = 200$, $R_B = 100\ k\Omega$, $R_C = 1\ k\Omega$ and $V_{BE} = 1.0$ V, The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively :

(1) 20μA and 3.5V (2) 25μA and 3.5V

(3) 25μA and 2.5V (4) 20μA and 2.8V

Ans (2)

Sol. At saturation, $V_{CE} = 0$

$V_{CE} = V_{CC} - I_C R_C$

$\Rightarrow I_C = \dfrac{V_{CC}}{R_C} = 5 \times 10^{-3}$ A

Given

$$\beta_{dc} = \frac{I_C}{I_B}$$

$$I_B = \frac{5 \times 10^{-3}}{200}$$

$I_B = 25\ \mu A$

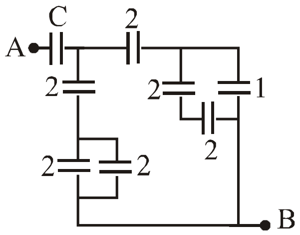
At input side

$V_{BB} = I_B R_B + V_{BE}$

$= (25mA)(100k\Omega) + 1V$

$V_{BB} = 3.5$ V

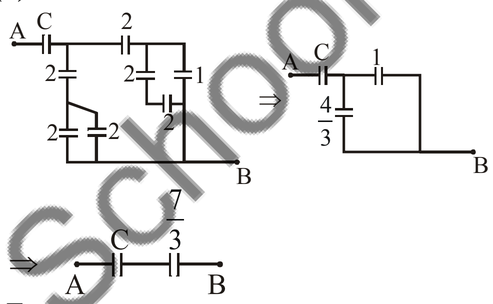
11. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.



(1) $\dfrac{7}{10}$μF (2) $\dfrac{7}{11}$μF (3) $\dfrac{6}{5}$μF (4) 4μF

Ans (2)

Sol.



From equs.

$$\frac{\frac{7C}{3}}{\frac{7}{3}+C} = \frac{1}{2}$$

$\Rightarrow$ 14 C = 7 + 3 C

$\Rightarrow$ C = $\dfrac{7}{11}$

12. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is:

(1) 2 (2) $\sqrt{\dfrac{1}{2}}$ (3) 1 (4) $\dfrac{1}{2}$

Ans (3)

Sol. Orbital velocity $V = \sqrt{\dfrac{GMe}{r}}$

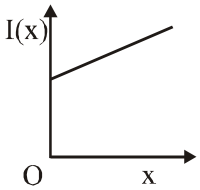
$$T_A = \frac{1}{2} m_A V_A^2$$

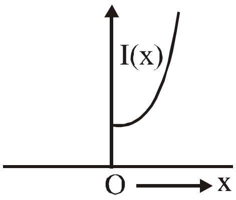
$$T_B = \frac{1}{2} m_B V_B^2$$

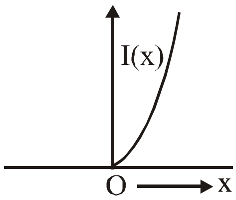
$$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{Gm}{R}}{2m \times \frac{Gm}{2R}}$$

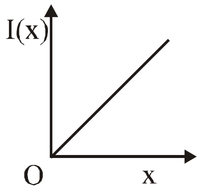
$$\Rightarrow \frac{T_A}{T_B} = 1$$

13. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is I(x)'. Which one of the graphs represents the variation of I(x) with x correctlv?

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. $I = \frac{2}{5}mR^2 + mx^2$

14. When a certain photosensistive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photo current is – $V_0/2$. When the surface is illuminated by monochromatic light of frequency ν/2, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is:

(1) $\frac{3\nu}{2}$ (2) 2ν (3) $\frac{4}{3}\nu$ (4) $\frac{5\nu}{3}$

Ans. **(1)**

Sol. $h\nu = W + \frac{V_0}{2}e$

$\frac{h\nu}{2} = W + V_0 e$

On solving we get $\Rightarrow h\nu_0 = \frac{3}{2}h\nu$

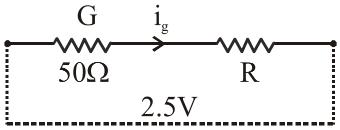
$\Rightarrow \nu_0 = \frac{3}{2}\nu$

15. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of:

(1) 6250 ohm (2) 250 ohm
(3) 200 ohm (4) 6200 ohm

Ans. **(3)**

Sol. $I_g = 4 \times 10^{-4} \times 25 = 10^{-2}$ A

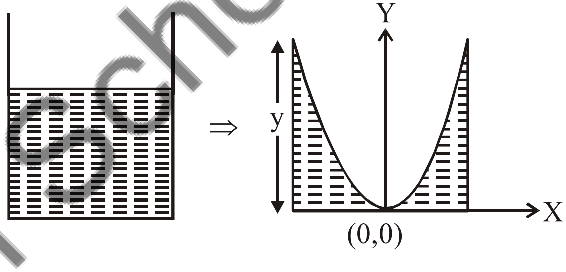


$2.5 = (50 + R)\ 10^{-2}\ \therefore\ R = 200\ \Omega$

16. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be:

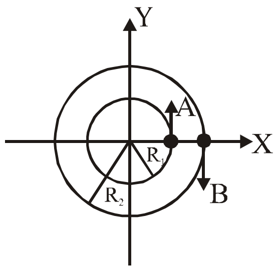
(1) 1.2 (2) 0.1 (3) 2.0 (4) 0.4

Ans. **(3)**

Sol. 

$y = \frac{\omega^2 x^2}{2g} = \frac{(2\times 2\pi)^2 \times (0.05)^2}{20} \simeq 2$ cm

17. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure :

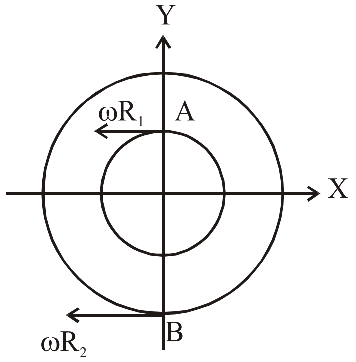


The relative velocity $\vec{\upsilon}_A - \vec{\upsilon}_B$ at $t = \frac{\pi}{2\omega}$ is given by :

(1) $-\omega(R_1 + R_2)\hat{i}$ (2) $\omega(R_1 + R_2)\hat{i}$
(3) $\omega(R_1 - R_2)\hat{i}$ (4) $\omega(R_2 - R_1)\hat{i}$

Ans. **(4)**

Sol. $\theta = \omega t = \omega\frac{\pi}{2\omega} = \frac{\pi}{2}$



$$\vec{V}_A - \vec{V}_S = \omega R_1\left(-\hat{i}\right) - \omega R_2\left(-i\right)$$

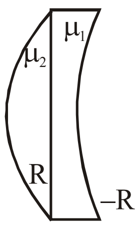
18. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be :

(1) $f_1 - f_2$ (2) $f_1 + f_2$

(3) $\frac{R}{\mu_2 - \mu_1}$ (4) $\frac{2f_1f_2}{f_1+f_2}$

Ans. **(3)**

Sol.



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1-\mu_1}{R} + \frac{\mu_2 - 1}{R}$$

19. Let ℓ, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be:

(1) [LTA] (2) [LA^{-2}] (3) [A^{-1}] (4) [LT^2]

Ans. **(3)**

Sol. $\left[\frac{\ell}{r}\right] = T$

[CV] = AT

So, $\left[\frac{\ell}{rCV}\right] = \frac{T}{AT} = A^{-1}$

20. In a radioactive decay chain, the initial nucleus is ${}^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus, If ${}^{A}_{Z}X$, A and Z are given by :

(1) A = 208; Z = 80

(2) A = 202; Z = 80

(3) A = 200; Z = 81

(4) A = 208; Z = 82

Ans. **(4)**

Sol. ${}^{232}_{90} \longrightarrow {}^{208}_{78} + {}^{4}_{2}(He) \times 6$

${}^{208}_{78}Y \longrightarrow {}^{208}_{82}X + 4\beta$ praticle

21. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to :

(1) 10^2T (2) 10^{-4}T

(3) 1T (4) 10^{-2}T

Ans. **(2)**

Sol. $I = \varepsilon_0\ C\ E^2_{rms}$ $\&E_{rms} = cB_{rms}$

$I = \varepsilon_0\ C^3\ B^2_{rms}$

$B_{rms} = \sqrt{\frac{I}{\in_0 C^3}}$ B_{rms} 10^{-4}

22. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to:

(1) $328ms^{-1}$ (2) $322ms^{-1}$

(3) $341ms^{-1}$ (4) $335ms^{-1}$

Ans. **(1)**

Sol. $\frac{\lambda_1}{4} = 11$ cm. + e $\Rightarrow \frac{v}{512\times4}$ = 11 cm. + e ….(1)

$\frac{\lambda_2}{4} = 27$ cm. + e $\Rightarrow \frac{v}{256\times4}$ = 27 cm. + e ….(2)

Equation (2) – (1) $\frac{v}{256\times4}\times\frac{1}{2} = 0.16$

v = 0.16 × 2 × 4 × 256 = 327.68 m/s

23. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to:

(1) 4×10^{-8}s (2) 3×10^{-6}s
(3) 2×10^{-7}s (4) 0.5×10^{-8}s

Ans. (1)

Sol. $t \propto \frac{\text{Volume}}{\text{velocity}}$

volume $\propto \frac{T}{P}$

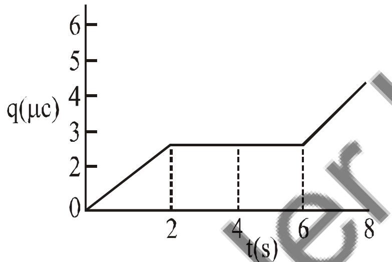
$\therefore \quad t \propto \frac{\sqrt{T}}{P}$

$$\frac{t_1}{6\times10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}}$$

$t_1 = 3.8 \times 10^{-8}$

$\approx 4 \times 10^{-8}$

24. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure:
What is the value of current at t = 4 s ?

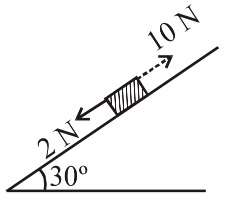


(1) 3μA (2) 2μA (3) zero (4) 1.5μA

Ans. (3)

Sol. Since $\left.\frac{dq}{dt}\right|_{t=4s} = 0$ Therefore current = 0

25. A block kept on a rough inclined plane, as shown in the figure, remaias at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction betwreen the block and the plane is :
[Take g = 10 m/s²]



(1) $\frac{2}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $2 + mg \sin30 = \mu mg \cos30°$

$10 = mg\sin 30 + \mu\ mg \cos30°$

$= 2\mu mg \cos30 - 2$

$6 = \mu mg \cos 30$

$4 = mg \sin 30$

$\frac{3}{2} = \ \times\sqrt{\ }$

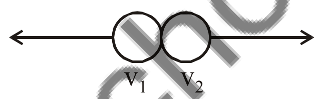
$\mu = \frac{\sqrt{3}}{2}$

26. An alpha-particle of mass m suffers 1-dimensional elastic coolision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :-

(1) 4 m (2) 3.5 m (3) 2 m (4) 1.5 m

Ans. (1)

Sol.



$mv_0 = mv_2 - mv_1$

$$\frac{1}{2}mV_1^2 = 0.36 \times \frac{1}{2}mV_0^2$$

$v_1 = 0.6v_0$

$$\frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mV_0^2$$

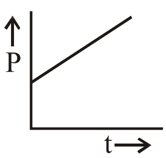
$$V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0$$

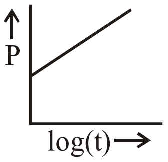
$mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$

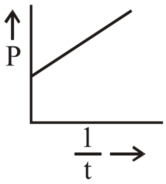
$\Rightarrow$ 1.6m = $0.8\sqrt{mM}$

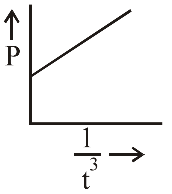
$4m^2 = mM$

27. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by :-

(1)


(2)


(3)


(4)


Ans. (4)

Sol. $V = ct \Rightarrow \frac{4}{3}\pi r^3 = ct \Rightarrow r = kt^{1/3}$

$$P = P_0 + \frac{4T}{kt^{1/3}} \Rightarrow P = P_0 + c\left(\frac{1}{t^{1/3}}\right)$$

28. To double the coverging range of a TV transmittion tower, its height should be multiplied by :-

(1) $\frac{1}{\sqrt{2}}$ (2) 4 (3) $\sqrt{2}$ (4) 2

Ans. (2)

Sol. Range $= \sqrt{2hR}$

To double the range h have to be made 4 times

29. A parallel plate capacitor with plates of area $1m^2$ each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge each plate is :-

(Take $\varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N-m^2}$)

(1) 7.85×10^{-10} C (2) 6.85×10^{-10} C
(3) 9.85×10^{-10} C (4) 8.85×10^{-10} C

Ans. (4)

Sol. $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$Q = AE\epsilon_0$

$Q = (1)(100)(8.85\times10^{-12})$

$Q = 8.85 \times 10^{-10}C$

30. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to :-

(1) 2020 nm (2) 220 nm
(3) 250 nm (4) 1700 nm

Ans. (3)

Sol. $\lambda = \frac{1240}{5.6-0.7}$ nm

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Saturday 12th JANUARY, 2019)   TIME : 02 : 30 PM To 05 : 30 PM
CHEMISTRY

1. An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) in it has escaped from the vessel. Assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is
   (1) 750 °C
   (2) 750 K
   (3) 500 °C
   (4) 500 K

Answer (4)

Sol. Initial number of moles of an ideal gas = n_1

Find number of moles of the ideal gas

$$= n_2 = n_1 - \frac{2n_1}{5} = \frac{3n_1}{5}$$

At constant volume and pressure, the number of moles of an ideal gas is inversely proportional to temperature

$$n \propto \frac{1}{T}$$

$$n_1T_1 = n_2T_2$$

$$T_2 = \frac{n_1}{n_2}T_1 = \frac{5}{3} \times 300 = 500\,\text{K}$$

2. Given
   (i) C (graphite) + $O_2(g) \rightarrow CO_2(g)$;
   $\Delta rH^{\ominus} = x$ kJ mol^{-1}
   (ii) C (graphite) + $\frac{1}{2}O_2(g) \rightarrow CO_2(g)$;
   $\Delta rH^{\ominus} = y$ kJ mol^{-1}
   (iii) $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$;
   $\Delta rH^{\ominus} = z$ kJ mol^{-1}

   Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct?
   (1) $x = y - z$
   (2) $x = y + z$
   (3) $y = 2z - x$
   (4) $z = x + y$

Answer (2)

Sol. According to Hess's law, the enthalpy change of a reaction does not depend on the number of steps involved in the reaction.

$$C(\text{graphite}) + \frac{1}{2}O_2(g) \rightarrow CO\,(g)\ \Delta H_1^\circ = y\ \text{kJ mol}^{-1}$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)\ \Delta H_2^\circ = z\ \text{kJ mol}^{-1}$$

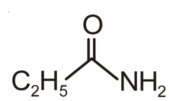
$$C(\text{graphite}) + O_2(g) \rightarrow \Delta^\circ H_3 = x\ \text{kJ mol}^{-1}$$

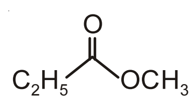
$\because\ \Delta H_3^\circ = \Delta H_1^\circ + \Delta H_2^\circ$

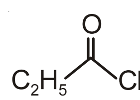
$x = y + z$

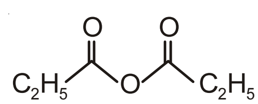
** in reaction ii, Product should be CO (gas) instead of CO_2 (gas).

3. The increasing order of the reactivity of the following with $LiAlH_4$ is

   (A) 

   (B) 

   (C) 

   (D) 

   (1) (A) < (B) < (C) < (D)
   (2) (B) < (A) < (D) < (C)
   (3) (A) < (B) < (D) < (C)
   (4) (B) < (A) < (C) < (D)

Answer (3)

Sol. The reactivity order of carboxylic acid derivatives depends on the leaving tendency of the leaving group. Higher the leaving tendency of the leaving group, higher will be the reactivity of the compound. Therefore, reactivity order towards $LiAlH_4$ is

Acid halide > Acid anhydride > Ester > Amide

4. Among the following, the false statement is

(1) Tyndall effect can be used to distinguish between a colloidal solution and a true solution.

(2) Latex is a colloidal solution of rubber particles which are positively charged

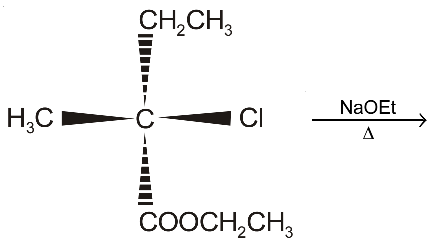
(3) Lyophilic sol can be coagulated by adding an electrolyte.

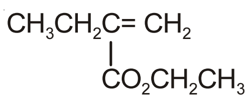
(4) It is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane.

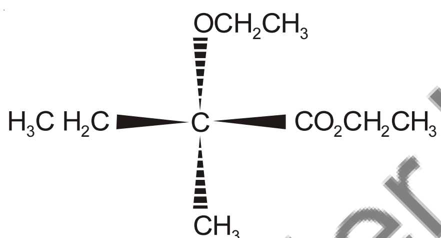
Answer (2)

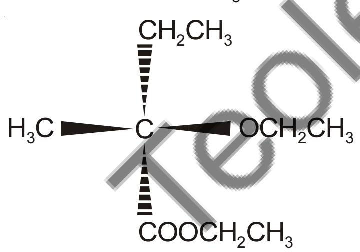
Sol. Latex is colloidal solution of rubber particles which are negatively charged.

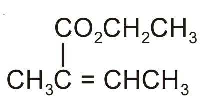
5. The major product of the following reaction is



(1) 

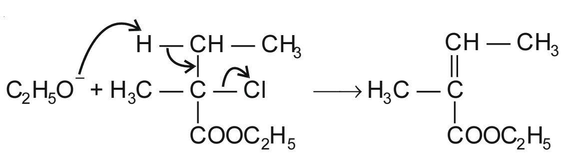
(2) 

(3) 

(4) 

Answer (4)

Sol. High temperature and strong base favours elimination reaction forming more stable alkene according to Saytzeff rule.



6. The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 BM. The suitable ligand for this complex is

(1) CO (2) Ethylenediamine

(3) NCS^- (4) CN^-

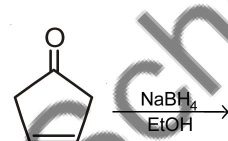
Answer (3)

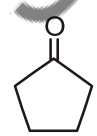
Sol. Electronic configuration of Mn^{2+} is

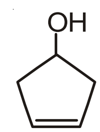
Mn^{+2} : $3d^5$

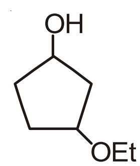
It has 5 unpaired electrons which corresponds to magnetic moment of $\sqrt{35}$ = 5.9 BM. This shows that the homoleptic complex of Mn^{2+} has only weak field ligands and that is NCS^-. The remaining three ligands are strong field ligands.

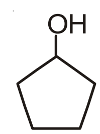
7. The major product of the following reaction is



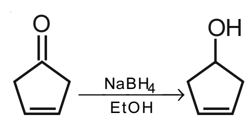
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

$NaBH_4$ does not reduces the double bond in β-γ unsaturated aldehydes/ ketones.

Only the keto group will be reduced.

8. If K_{sp} of Ag_2CO_3 is 8×10^{-12}, the molar solubility of Ag_2CO_3 in 0.1 M $AgNO_3$ is

(1) 8×10^{-11} M

(2) 8×10^{-12} M

(3) 8×10^{-13} M

(4) 8×10^{-10} M

Answer (4)

Sol. $AgNO_3 \longrightarrow \underset{0.1}{Ag^+} + \underset{0.1}{NO_3^-}$

$$Ag_2CO_3 \rightleftharpoons \underset{\substack{0.1+2x \\ \approx 0.1}}{2Ag^+} + \underset{x}{CO_3^{2-}}$$

$$K_{sp} = [Ag^+]^2[CO_3^{2-}]$$

$$= (0.1)^2\ x = 8 \times 10^{-12}$$

$$0.01\ x = 8 \times 10^{-12}$$

$$x = 8 \times 10^{-10}\ M$$

9. $\overset{\circ}{\wedge}_m$ for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^{2}mol^{-1}, respectively. If the conductivity of 0.001 M HA is 5×10^{-5} S cm^{-1}, degree of dissociation of HA is

(1) 0.25

(2) 0.125

(3) 0.50

(4) 0.75

Answer (2)

Sol. Λ°_m (NaCl) = 126.4 S cm^2 mol^{-1}

Λ°_m (HCl) = 425.9 S cm^2 mol^{-1}

Λ°_m (NaA) = 100.5 S cm^2 mol^{-1}

Λ°_m (HA) = 425.9 – 126.4 + 100.5 = 400 S cm^2 mol^{-1}

K(HA) = 5×10^{-5} S cm^{-1}

$$\Lambda^c{}_m = \frac{K \times 1000}{\text{Molarity}} = \frac{5 \times 10^{-5} \times 1000}{0.001} = 50$$

$$\alpha = \frac{\Lambda^c{}_m}{\Lambda^\circ{}_m} = \frac{50}{400} = 0.125$$

10. The major product of the following reaction is

$$CH_3CH_2\underset{Br}{CH}\underset{Br}{CH_2} \xrightarrow[\text{(ii) NaNH}_2\text{ in liq NH}_3]{\text{(i) KOH alc.}}$$

(1) $CH_3CH_2C \equiv CH$

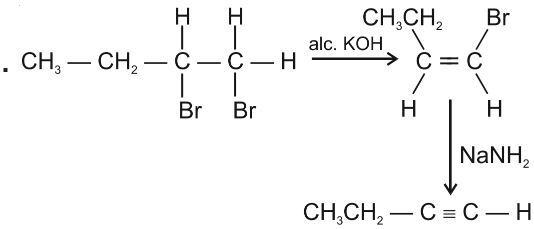
(2) $CH_3CH = CHCH_2NH_2$

(3) $CH_3CH_2\underset{NH_2}{CH} - \underset{NH_2}{CH_2}$

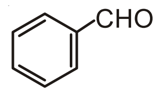
(4) $CH_3CH = C = CH_2$

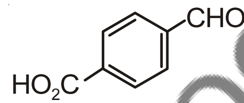
Answer (1)

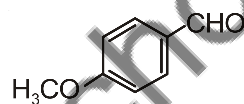
Sol.

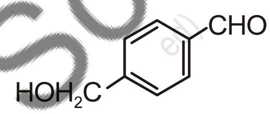


11. The aldehydes which will **not** form Grignard product with one equivalent Grignard reagent are

(A) 

(B) 

(C) 

(D) 

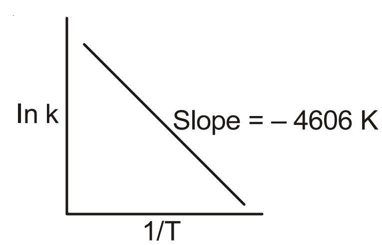
(1) (B), (C) (2) (B), (D)

(3) (B), (C), (D) (4) (C), (D)

Answer (2)

Sol. Grignard reagent will not react with aldehydes if it has a functional group which contains acidic hydrogen. Options (B) and (D) have —COOH and —CH_2OH respectively which contan acidic H-atom.

12. For a reaction, consider the plot of ln k versus 1/T given in the figure. If the rate constant of this reaction at 400 K is 10^{-5} s^{-1}, then the rate constant at 500 K is



(1) 4×10^{-4} s^{-1}

(2) 10^{-6} s^{-1}

(3) 2×10^{-4} s^{-1}

(4) 10^{-4} s^{-1}

Answer (4)

Sol. $\ln K = \ln A - \frac{E_a}{RT}$

$$\text{Slope} = \frac{E_a}{R} = 4606\,K$$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$$= \frac{4606(100)}{400 \times 500}$$

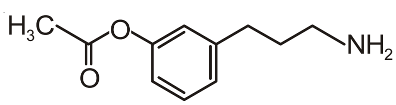
$$= 2.303$$

$$\Rightarrow \log\left(\frac{K_2}{K_1}\right) = 1$$

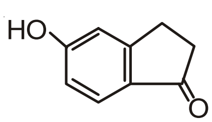
$$\frac{K_2}{K_1} = 10$$

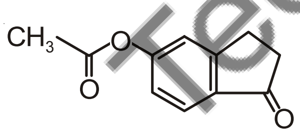
$$\Rightarrow K_2 = 10K_1 = 10^{-5} \times 10 = 10^{-4}\ S^{-1}$$

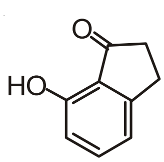
13. The major product of the following reaction is

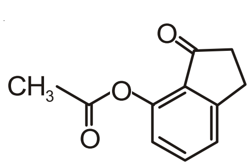


$$\xrightarrow[\text{(iii) } H_2SO_4\text{(conc.)},\Delta]{\text{(i) } NaNO_2/H^+ \quad \text{(ii) } CrO_3/H^+}$$

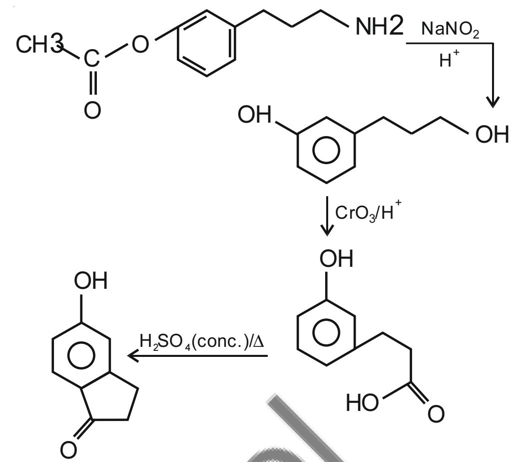
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

14. The compound that is NOT a common component of photochemical smog is:

(1) $H_3C-\underset{\underset{O}{\|}}{C}-OONO_2$

(2) $CH_2 = CHCHO$

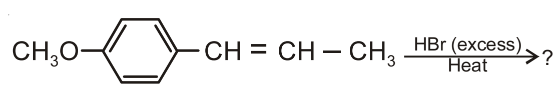
(3) CF_2Cl_2

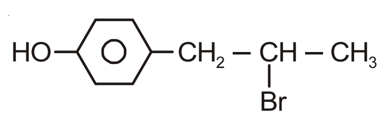
(4) O_3

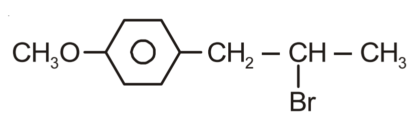
Answer (3)

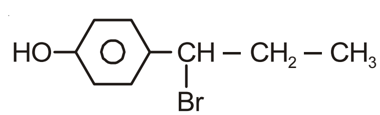
Sol. CF_2Cl_2 is not a common component of photochemical smog.

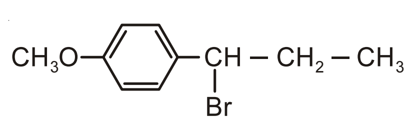
15. The major product in the following conversion is



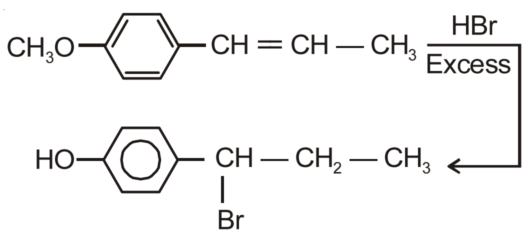
(1) 

(2) 

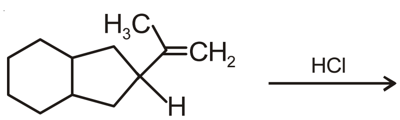
(3) 

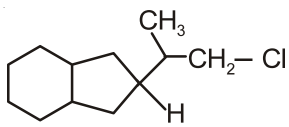
(4) 

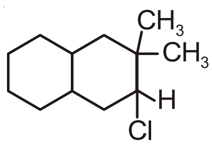
Answer (3)

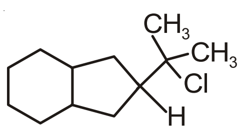
Sol. 

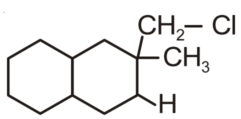
16. The major product of the following reaction is



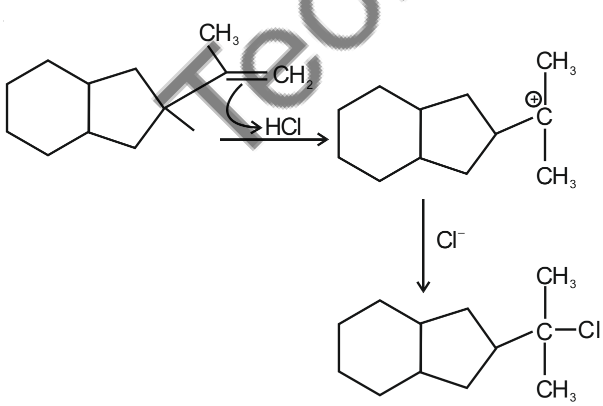
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

17. Molecules of benzoic acid (C_6H_5COOH) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2 K. If the percentage association of the acid to form dimer in the solution is 80, then w is (Given that K_f = 5 K kg mol^{-1}, Molar mass of benzoic acid = 122 g mol^{-1})

(1) 1.5 g

(2) 2.4 g

(3) 1.8 g

(4) 1.0 g

Answer (2)

Sol. $2C_6H_5COOH \longrightarrow (C_6H_5COOH)_2$

t = 0	1	0
t	$1 - 2\alpha$	α

Moles at equilibrium = $1 - 2\alpha + \alpha = 1 - \alpha$

$2\alpha = 0.8$, $\alpha = 0.4$

Moles at equilibrium = 0.6

i = 0.6

$$\Delta T_f = ik_f m \Rightarrow \quad 2 = 0.6 \times 5 \times \left(\frac{\frac{w}{122}}{30}\right) \times 1000$$

w = 2.4 g

18. Chlorine on reaction with hot and concentrated sodium hydroxide gives

(1) Cl^- and ClO^-

(2) Cl^- and ClO_2^-

(3) ClO_3^- and ClO_2^-

(4) Cl^- and ClO_3^-

Answer (4)

Sol. $3Cl_2 + 6NaOH \longrightarrow 5NaCl + NaClO_3 + 3H_2O$

19. The correct statement(s) among I to III with respect to potassium ions that are abundant within the cell fluids is/are

I. They activate many enzymes

II. They participate in the oxidation of glucose to produce ATP

III. Along with sodium ions, they are responsible for the transmission of nerve signals

(1) I and III only

(2) I, II and III

(3) III only

(4) I and II only

Answer (2)

Sol. K^+ ions act as carriers for nerve signals, are activators for many enzymes and participate in the oxidation of glucose to form ATP.

20. If the de Broglie wavelength of the electron in n^{th} Bohr orbit in a hydrogenic atom is equal to $1.5\ \pi a_0$ (a_0 is Bohr radius), then the value of n/z is

(1) 0.40 (2) 1.50

(3) 0.75 (4) 1.0

Answer (3)

Sol. $n\lambda = 2\pi r$

$$r = a_0 \frac{n^2}{z}$$

$$n\lambda = \frac{2\pi a_0 n^2}{z}$$

$$\lambda = \frac{2\pi a_0 n^2}{z}$$

$$1.5\pi a^0 = 2\pi a_0 \frac{n}{2}$$

$$\frac{n}{z} = \frac{3}{4} = 0.75$$

21. The volume strength of 1M H_2O_2 is

(Molar mass of H_2O_2 = 34 g mol^{-1})

(1) 11.35

(2) 22.4

(3) 5.6

(4) 16.8

Answer (1)

Sol. Volume strength ≈ 11.2 × M

≈ 11.2

22. The correct order of atomic radii is

(1) Ce > Eu > Ho > N (2) N > Ce > Eu > Ho

(3) Eu > Ce > Ho > N (4) Ho > N > Eu > Ce

Answer (3)

Sol. Atomic radii follows the order

Eu	>	Ce	>	Ho	>	N
199 pm		183 pm		176 pm		70 pm

23. The element that does NOT show catenation is

(1) Sn

(2) Ge

(3) Pb

(4) Si

Answer (3)

Sol. Lead Pb

24. The two monomers for the synthesis of nylon 6, 6 are

(1) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_4NH_2$

(2) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_6NH$

(3) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_6NH_2$

(4) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_4NH_2$

Answer (3)

Sol. Monomer of Nylon–6, 6 are adipic acid and hexamethylene diammine.

25. The pair that does NOT require calcination is

(1) Fe_2O_3 and $CaCO_3 \cdot MgCO_3$

(2) $ZnCO_3$ and CaO

(3) ZnO and MgO

(4) ZnO and $Fe_2O_3 \cdot xH_2O$

Answer (3)

Sol. ZnO and MgO

They are oxides while other are carbonates or hydrated oxides which require calcination.

26. The upper stratosphere consisting of the ozone layer protects us from the sun's radiation that falls in the wavelength region of

(1) 200 – 315 nm (2) 600 – 750 nm

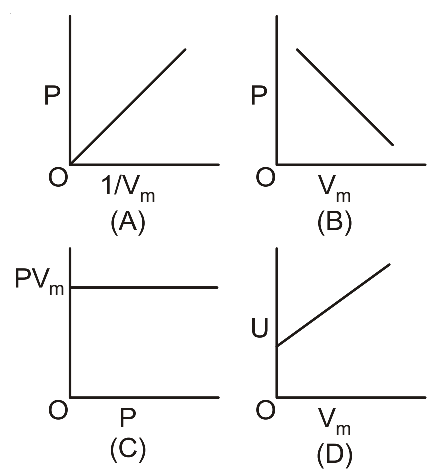
(3) 400 – 550 nm (4) 0.8 – 1.5 nm

Answer (1)

Sol. Ozone layer protects from ultra violet radiation.

∴ Wavelength range lies in 200 – 315 nm

27. The combination of plots which does not represent isothermal expansion of an ideal gas is



(1) (A) and (C) (2) (A) and (D)

(3) (B) and (C) (4) (B) and (D)

Answer (4)

Sol. (B) and (D) are not correct representation for isothermal expansion of ideal gas.

28. 8 g of NaOH is dissolved in 18 g of H_2O. Mole fraction of NaOH in solution and molality (in mol kg^{-1}) of the solution respectively are

(1) 0.2, 22.20

(2) 0.167, 22.20

(3) 0.167, 11.11

(4) 0.2, 11.11

Answer (3)

Sol. Mole faction $= \dfrac{n_2}{n_2 + n_1} = \dfrac{\frac{1}{5}}{\frac{1}{5}+1} = 0.167$

$n_2 = \dfrac{8}{40} \quad n_1 = \dfrac{18}{18}$

Molality $= \dfrac{8}{40} \times \dfrac{1000}{18} = 11.11\,\text{m}$

29. The element that shows greater ability to form $p\pi - p\pi$ multiple bonds, is

(1) Sn

(2) Si

(3) Ge

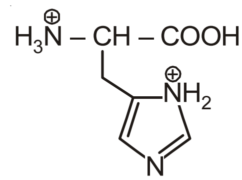
(4) C

Answer (4)

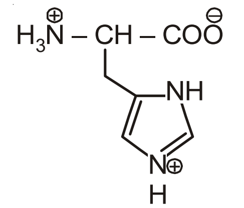
Sol. Carbon has small size so effective, lateral overlapping between 2p and 2p.

30. The correct structure of histidine in a strongly acidic solution (pH = 2) is

(1)



(2)



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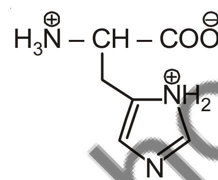
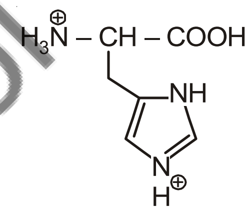
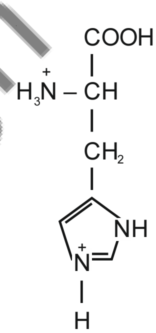


(4)



Answer (4)

Sol. Histidine (in strongly acidic solution)



TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let Z be the set of integers. If $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is :

(1) 2^{18} (2) 2^{10} (3) 2^{15} (4) 2^{12}

Ans (3)

Sol. A = {x ∈ z : $2^{(x+2)(x^2-5x+6)} = 1$}

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

A = {–2, 2, 3}

B = {x ∈ Z : –3 < 2x – 1 < 9}

B = {0, 1, 2, 3, 4}

A × B has is 15 elements so number of subsets of A × B is 2^{15}.

2. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then cos(α + β) – cos(α – β) is equal to :

(1) 0 (2) $-\sqrt{2}$ (3) –1 (4) $\sqrt{2}$

Ans (2)

Sol. A.M. ≥ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha . 4\cos^4\beta . 1.1)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^2\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

given that $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$

⇒ A.M. = G.M. ⇒ $\sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1$, $\cos\beta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\beta = \frac{1}{\sqrt{2}}$ as $\beta \in [0, \pi]$

cos (α + β) – cos (α – β) = –2 sin α sin β

$= -\sqrt{2}$

3. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, x – 2y – kz = 3 is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is:

(1) $-\frac{5}{3}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{3}{5}$

Ans (3)

Sol. DR's of line are 2, 1, –2

normal vector of plane is $\hat{i} - 2\hat{j} - k\hat{k}$

$$\sin\alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha = \frac{2k}{3\sqrt{k^2+5}} \quad ....(1)$$

$$\cos\alpha = \frac{2\sqrt{2}}{3} \quad ....(2)$$

$(1)^2 + (2)^2 = 1 \Rightarrow k^2 = \frac{5}{3}$

4. If a straight line passing thourgh the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

(1) x – y + 7 = 0

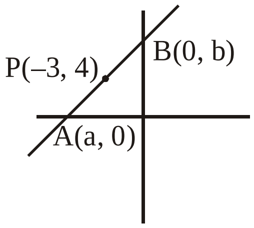
(2) 3x – 4y + 25 = 0

(3) 4x + 3y = 0

(4) 4x – 3y + 24 = 0

Ans (4)

Sol.



Let the line be $\frac{x}{a}+\frac{y}{b}=1$

$(-3, 4) = \left(\frac{a}{2},\frac{b}{2}\right)$

$a = -6,\ b = 8$

equation of line is $4x - 3y + 24 = 0$

5. The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

(2) $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

(3) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

(4) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

Ans (2)

Sol. $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$

$$\int \frac{\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}$$

Let $\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right) = t$

$$-\frac{1}{2}\int\frac{dt}{t^4}=\frac{1}{6t^3}+C \Rightarrow \frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

6. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 9 (2) 11 (3) 12 (4) 7

Ans (3)

Sol. Let m-men, 2-women

${}^mC_2 \times 2 = {}^mC_1\ {}^2C_1 \cdot 2 + 84$

$m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$

$m = 12$

7. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a\in R$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x)-14}{(x-1)^2}=0\ (x\neq 1)$ is :

(1) 6 (2) 5 (3) 7 (4) –7

Ans (3)

Sol. $f'(x) = 3x^2 - 6(a - 2)x + 3a$

$f'(x) \geq 0\ \forall\ x \in (0, 1]$

$f'(x) \leq 0\ \forall\ x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x - 1)^2 (x - 7)$

$\frac{f(x)-14}{(x-1)^2} = x - 7$

8. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x\in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to :

(1) 4e (2) $4e^2$ (3) 2e (4) $2e^2$

Ans (1)

Sol. $\frac{f'(x)}{f(x)} = 1\ \forall\ x \in R$

Intergrate & use $f(1) = 2$

$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))\ f'(x)$

$h'(1) = f'(f(1))\ f'(1)$

$= f'(2)\ f'(1)$

$= 2e \cdot 2 = 4e$

9. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point.

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(3) $\left(-\frac{1}{8}, 7\right)$ (4) $\left(\frac{1}{8}, -7\right)$

Ans (4)

Sol. $y = x^2 - 5x + 5$

$$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$$

$$\text{at } x = \frac{7}{2},\ y = \frac{-1}{4}$$

Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is $2x - y - \frac{29}{4} = 0$

Now check options

$$x = \frac{1}{8},\ y = -7$$

10. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to :

(1) $\{\sqrt{3}\}$ (2) $\{\sqrt{3}, -\sqrt{3}\}$

(3) $\{1, -1\}$ (4) $\{3, -3\}$

Ans (2)

Sol. All four points are coplaner so

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$$(\lambda^2 + 1)^2 (3 - \lambda^2) = 0$$

$$\lambda = \pm\sqrt{3}$$

11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

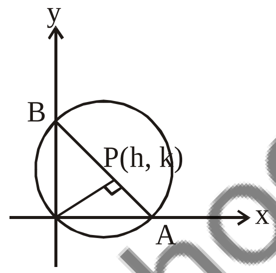
(2) $(x^2 + y^2)(x + y) = R^2xy$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

Ans (3)

Sol.



Slope of AB $= \frac{-h}{k}$

Equation of AB is $hx + ky = h^2 + k^2$

$$A\left(\frac{h^2+k^2}{h}, 0\right),\quad B\left(0, \frac{h^2+k^2}{k}\right)$$

$AB = 2R$

$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$

12. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

(1) $x = y\cot\theta + 2\tan\theta$ (2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$ (4) $y = x\tan\theta + 2\cot\theta$

Ans (1)

Sol. $x^2 = 8y$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$$

$\therefore\ x_1 = 4\tan\theta$

$y_1 = 2\tan^2\theta$

Equation of tangent :-

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

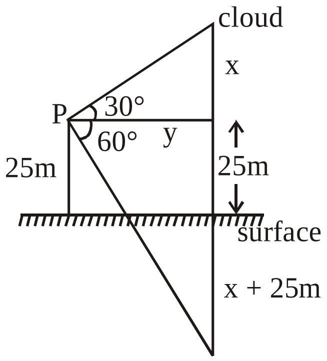
$\Rightarrow x = y\cot\theta + 2\tan\theta$

13. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30º and the angle of depression of reflection of the cloud in the lake from P be 60º, then the height of the cloud (in meters) from the surface of the lake is :

(1) 42 (2) 50 (3) 45 (4) 60

Ans (2)

Sol.



$$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}\,x \qquad .....(i)$$

$$\tan 60° = \frac{25 + x + 25}{y}$$

$\Rightarrow \sqrt{3}\,y = 50 + x$

$\Rightarrow 3x = 50 + x$

$\Rightarrow x = 25$ m

$\therefore$ Height of cloud from surface = 25 + 25 = 50m

14. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$ is equal to :

(1) $\frac{1}{2} - e - \frac{1}{e^2}$ (2) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

(3) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$ (4) $\frac{3}{2} - e - \frac{1}{2e^2}$

Ans. (4)

$$\int_1^e \left(\frac{x}{e}\right)^{2x} \log_e x.dx - \int_1^e \left(\frac{e}{x}\right)\log_e x.dx$$

Let $\left(\frac{x}{e}\right)^{2x} = t$, $\left(\frac{e}{x}\right)^{x} = v$

$$= \frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1} dt + \int_e^1 dv$$

$$= \frac{1}{2}\left(1 - \frac{1}{e^2}\right) + (1 - e) = \frac{3}{2} - \frac{1}{2e^2} - e$$

15. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + ... + \frac{1}{5n}\right)$ is equal to :

(1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$

(3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$

Ans. (2)

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)} = \int_0^2 \frac{dx}{1+x^2} = \tan^{-1} 2$$

16. The set of all values of λ for which the system of linear equations.

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

Ans. (2)

$$\begin{vmatrix} \lambda - 1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda - 1)^3 = 0 \Rightarrow \lambda = 1$$

17. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be:

(1) 14 (2) 11 (3) 9 (4) 12

Ans. (1)

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

$$2.\frac{n!}{5!\,(n-5)!} = \frac{n!}{4!\,(n-4)!} + \frac{n!}{6!\,(n-6)!}$$

$$\frac{2}{5}.\frac{1}{n-5} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

n = 14 satisfying equation.

18. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to :

(1) 60º (2) 30º (3) 90º (4) 45º

Ans. (2)

$$\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right).\vec{c}=\frac{1}{2}\vec{b}$$

$\because$ $\vec{b}$ & $\vec{c}$ are linearly independent

$\therefore$ $\vec{a}.\vec{c}=\frac{1}{2}$ & $\vec{a}.\vec{b}=0$

(All given vectors are unit vectors)

$\therefore$ $\vec{a}\wedge\vec{c}=60°$ & $\vec{a}\wedge\vec{b}=90°$

$\therefore$ $|\alpha-\beta| = 30°$

19. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det(A) lies in the interval :

(1) $\left[\frac{5}{2},4\right)$ (2) $\left(\frac{3}{2},3\right]$

(3) $\left(0,\frac{3}{2}\right]$ (4) $\left(1,\frac{5}{2}\right]$

Ans (2)

$$|A| = \begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$$

$= 2(1 + \sin^2\theta)$

$$\theta \in \left(\frac{3\pi}{4},\frac{5\pi}{4}\right) \Rightarrow \frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le \sin^2\theta < \frac{1}{2}$$

$\therefore$ $|A| \in [2, 3)$

20. $\lim\limits_{x\to1-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ ie equal to :

(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\frac{\pi}{2}}$ (3) $\sqrt{\frac{2}{\pi}}$ (4) $\sqrt{\pi}$

Ans. (3)

$$\lim_{x\to1^-}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}\times\frac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$$

$$\lim_{x\to1^-}\frac{2\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)}$$

$$\lim_{x\to1^-}\frac{2\cos^{-1}x}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{\pi}}$$

Put $x = \cos\theta$

$$\lim_{\theta\to0^+}\frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}\cdot\frac{1}{2\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}$$

21. The expression $\sim(\sim p\to q)$ is logically equvalent to :

(1) $\sim p \wedge \sim q$ (2) $p \wedge q$

(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

Ans. (1)

p	q	~p	~p→q	~(~p→q)	(~p ^ ~q)
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

22. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is :

(1) 55 (2) 49 (3) 48 (4) 54

Ans. (4)

General term $T_{r+1} = {}^{60}C_r\ 7^{\frac{60-r}{5}}\ 3^{\frac{r}{10}}$

$\therefore$ for rational term, r = 0, 10, 20, 30, 40, 50, 60

$\Rightarrow$ no of rational terms = 7

$\therefore$ number of irrational terms = 54

23. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

(1) 1 (2) 3 (3) 7 (4) 5

Ans. (3)

mean $\bar{x} = 4, \sigma^2 = 5.2, n = 5,. x_1 = 3\ x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9$ ....(i)

$\frac{\sum x_i^2}{x} - (\bar{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65$ ....(ii)

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

24. If the sum of the first 15 tems of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to :

(1) 9 (2) 27 (3) 108 (4) 54

Ans. (2)

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + ...15\text{ term}$$

$$= \frac{27}{64}\sum_{r=1}^{15} r^3$$

$$= \frac{27}{64}\cdot\left[\frac{15(15+1)}{2}\right]^2$$

= 225 K (Given in question)

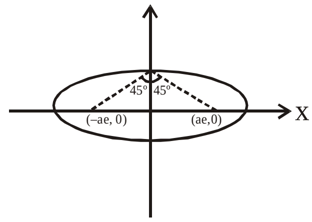
K = 27

25. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS)$ = 8 sq. units, then the length of a latus rectum of the ellipse is :

(1) $2\sqrt{2}$ (2) 2 (3) 4 (4) $4\sqrt{2}$

Ans. (3)

$m_{SB} \cdot m_{S'B} = -1$



$b^2 = a^2e^2$ .... (i)

$\frac{1}{2} S'B \cdot SB = 8$

$S'B. SB = 16$

$a^2e^2 + b^2 = 16$ ..... (ii)

$b^2 = a^2(1 - e^2)$ ..... (iii)

using (i),(ii), (iii) a = 4

$b = 2\sqrt{2}$

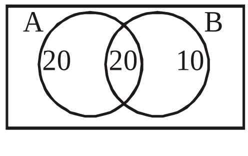
$e = \frac{1}{\sqrt{2}}$

$\therefore \ell\ (L.R) = \frac{2b^2}{a} = 4$ $\boxed{\text{Ans.3}}$

26. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

(1) $\frac{2}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $\frac{5}{6}$

Ans. (2)



A → opted NCC

B → opted NSS

$\therefore$ P (nither A nor B) = $\frac{10}{60} = \frac{1}{6}$

27. The number of integral values of m for which the quadratic expression.
$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is :

(1) 8 (2) 7 (3) 6 (4) 3

Ans. (2)

Exprsssion is always positve it

$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$

&

$D < 0 \Rightarrow m^2 - 6m - 3 < 0$

$3-\sqrt{12} < m < 3+\sqrt{12}$ .... (iii)

$\therefore$ Common interval is

$3-\sqrt{12} < m < 3+\sqrt{12}$

$\therefore$ Intgral value of m {0,1,2,3,4,5,6}

28. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/ loss (in rupees) is :

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss

(3) 0 (4) $\frac{400}{9}$ loss

Ans. (3)

Expected Gain/ Loss =
= w × 100 + Lw (–50 + 100) + L^2w (–50 –50 + 100) + L^3 (–150)

$= \frac{1}{3}\times 100+\frac{2}{3}.\frac{1}{3}(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0) + \left(\frac{2}{3}\right)^3(-150) = 0$

here w denotes probability that outcome 5 or 6 ($w = \frac{2}{6}=\frac{1}{3}$)

here L denotes probability that outcome 1,2,3,4 ($L = \frac{4}{6}=\frac{2}{3}$)

29. If a curve passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :

(1) $\left(-\sqrt{2},1\right)$ (2) $\left(\sqrt{3},0\right)$

(3) (–1, 2) (4) (3, 0)

Ans. (2)

$\frac{dy}{dx}=\frac{x^2-2y}{x}$ (Given)

$\frac{dy}{dx}+2\frac{y}{x}=x$

I.F = $e^{\int\frac{2}{x}dx} = x^2$

$\therefore$ $y.x^2 = \int x.x^2 dx + C$

$= \frac{x^4}{y}+C$

hence bpasses through (1, –2) $\Rightarrow$ C = $-\frac{9}{4}$

$\therefore$ $yx^2 = \frac{x^4}{4} - \frac{9}{4}$

Now check option(s) , Which is satisly by option (ii)

30. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2–3–4i|=4$. Then the minimum value of $|Z_1–Z_2|$ is :

(1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2

Ans. (1)

$|z_1| = 9$, $|z_2-(3+4i)| = 4$

C_1 (0, 0) radius $r_1 = 9$
C_2(3, 4), radius $r_2 = 4$
$C_1C_2 = |r_1–r_2| = 5$
$\therefore$ Circle touches internally

$\therefore$ $|z_1-z_2|_{min} = 0$