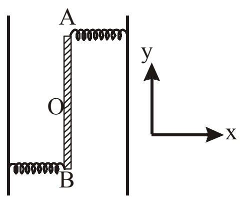
TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM

PHYSICS

1. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

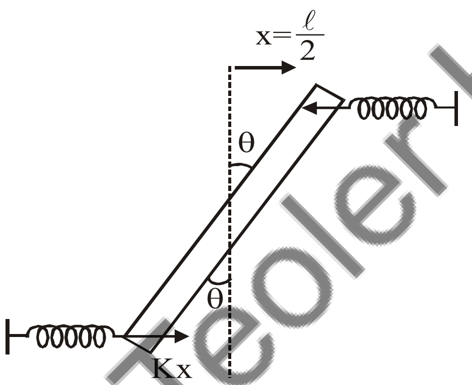
(2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Ans. **(1)**

Sol.



$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

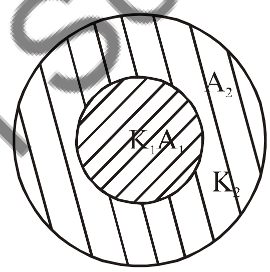
2. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

(1) $K_1 + K_2$ (2) $\frac{K_1 + K_2}{2}$

(3) $\frac{2K_1 + 3K_2}{5}$ (4) $\frac{K_1 + 3K_2}{4}$

Ans. **(4)**

Sol.



$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

$$= \frac{K_1\left(\pi R^2\right) + K_2\left(3\pi R^2\right)}{4\pi R^2}$$

$$= \frac{K_1 + 3K_2}{4}$$

3. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3}\sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave?

The wave is propagating along the

(1) negative x-axis with speed $25 ms^{-1}$

(2) The wave is propagating along the positive x-axis with speed $25\ ms^{-1}$

(3) The wave is propagating along the positive x-axis with speed $100\ ms^{-1}$

(4) The wave is propagating along the negative x-axis with speed $100\ ms^{-1}$

Ans. **(1)**

Sol. y= a sin(ωt + kx)

⇒ wave is moving along –ve x-axis with speed

$$\nu = \frac{\omega}{K} \Rightarrow \nu = \frac{50}{2} = 25\text{m/sec.}$$

4. A straight rod of length L extends from x = a to x=L + a. The gravitational force is exerted on a point mass 'm' at x = 0, if the mass per unit length of the rod is A + Bx^2, is given by:

(1) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

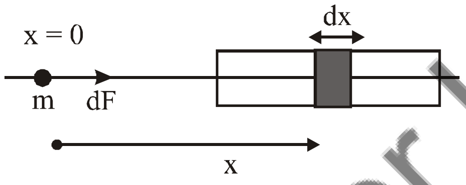
(2) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(3) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(4) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

Ans. (2)

Sol.



dm= (A + Bx^2)dx

$$dF = \frac{GM\,dm}{x^2}$$

$$= F = \int_a^{a+L} \frac{GM}{x^2}\left(A + Bx^2\right)dx$$

$$= GM\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$

$$= GM\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$$

5. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be:

(1) 10 V/m (2) 24 V/m

(3) 30 V/m (4) 6 V/m

Ans. (2)

Sol. $P_{refracted} = \frac{96}{100}P_I$

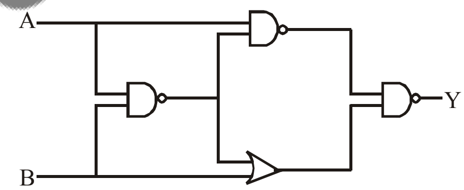
$$\Rightarrow K_2A_t^2 = \frac{96}{100}K_1A_i^2$$

$$\Rightarrow r_2A_t^2 = \frac{96}{100}r_1A_i^2$$

$$\Rightarrow A_t^2 = \frac{96}{100}\times\frac{1}{\frac{3}{2}}\times(30)^2$$

$$A_t\sqrt{\frac{64}{100}\times(30)^2} = 24$$

6. The output of the given logic circuit is :



(1) $\bar{A}B$

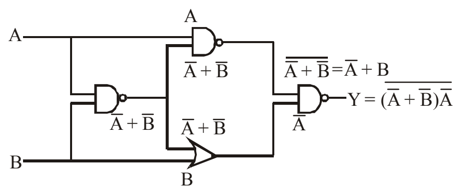
(2) $A\bar{B}$

(3) $AB + \overline{AB}$

(4) $A\bar{B} + \bar{A}B$

Ans. (2)

Sol.



$$Y = \overline{\left(\bar{A}+\bar{B}\right)\bar{A}}$$

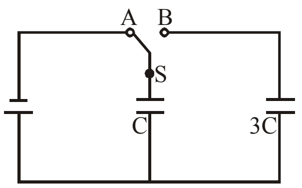
$$= \overline{\bar{A}+\bar{A}B}$$

$$= A\left(\overline{\bar{A}B}\right)$$

$$= A\left(A+\bar{B}\right)$$

$$= A + A\bar{B} \quad = A\bar{B}$$

7. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:



(1) $\frac{3}{8}\frac{Q^2}{C}$ (2) $\frac{3}{4}\frac{Q^2}{C}$ (3) $\frac{1}{8}\frac{Q^2}{C}$ (4) $\frac{5}{8}\frac{Q^2}{C}$

Ans. **(1)**

Sol. $V_i = \frac{1}{2}CE^2$

$$V_f = \frac{(CE)^2}{2\times 4c} = \frac{1}{2}\frac{CE^2}{4}$$

$$\Delta E = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2$$

8. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as:

(1) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$ (2) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$

(3) $r_n \propto n$, $E_n \propto n$ (4) $r_n \propto \sqrt{n}$, $E_n \propto n$

Ans. **(4)**

Sol. $F = \frac{dV}{dr} = kr = \frac{mv^2}{r}$

$mvr = \frac{nh}{2\pi}$

$r^2 \propto n$

$r^2 \propto \sqrt{n}$

$$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2 \propto r^2$$

$\propto n$

9. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then:

(1) P1 = 9 W, P_2 = 16 W

(2) P_1 = 4 W, P_2 = 16W

(3) P_1 = 16 W, P_2 = 4W

(4) P_1 16 W, P_2 = 9W

Ans. **(3)**

Sol. $R_1 = \frac{220^2}{25}$

$R_2 = \frac{220^2}{100}$

$L = \frac{220}{R_1 + R_2}$

$P_1 = i^2 R_1$

$P_2 = i^2$ $(R_2 = 4W)$

$$= \frac{220^2}{\left(\frac{220^2}{25} + \frac{220^2}{100}\right)} \times \frac{220^2}{25}$$

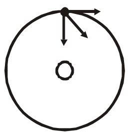
$$= \frac{400}{25} = 16W$$

10. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be :

(1) in a circular orbit of a different radius
(2) in the same circular orbit of radius R
(3) in an elliptical orbit
(4) such that it escapes to infinity

Ans. **(3)**

Sol. $m v\hat{i} + m v\hat{j}$



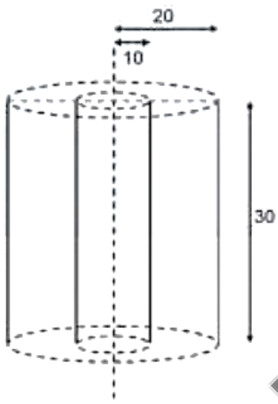
$= 2m\vec{v}^{1}$

$\vec{v} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$

11. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is:
(1) 12 cm (2) 18 cm (3) 16 cm (4) 14 cm

Ans. **(3)**

Sol.



$m\frac{\left(20^2+10^2\right)}{2} = mk^2$

$K = \sqrt{\frac{400+100}{2}}$

$K = \sqrt{250}$

$\Rightarrow K = 5\sqrt{10}$ cm

12. A passenger train of length 60m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction, and (ii) in the opposite directions is :

(1) $\frac{5}{2}$ (2) $\frac{25}{11}$ (3) $\frac{3}{2}$ (4) $\frac{11}{5}$

Ans. **(4)**

Sol. $t_1 = \frac{x}{v-u} = \frac{x}{50}$ (here total length of two trains is x)

$t_2 = \frac{x}{v+u} = \frac{x}{110}$

$\frac{t_1}{t_2} = \frac{11}{5}$

13. An ideal gas occupies a volume of $2m^3$ at a pressure of 3×10^6 Pa. The energy of the gas is:
(1) 3×10^2 (2) 10^8 J
(3) 6×10^4 J (4) 9×10^6 J

Ans. **(4)**

Sol. Energy $= \frac{1}{2}nRT = \frac{f}{2}PV$

$= \frac{f}{2}\left(3\times10^6\right)(2)$

$= f \times 3 \times 10^6$

Considering gas is monoatomic i.e. f = 3

$E = 9 \times 10^6$ J

Option-(4)

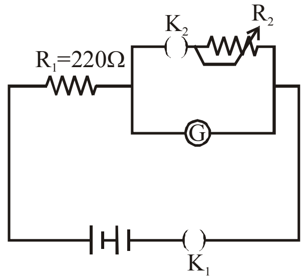
14. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?
(1) 0.6 (2) 0.5 (3) 0.3 (4) 0.4

Ans. **(1)**

Sol. $E_m + E_c = 160$
$E_m + 100 = 160$
$E_m = 60$

$\mu = \frac{E_m}{E_C} = \frac{60}{100}$
$\mu = 0.6$

15. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]:



(1) 12Ω (2) 25Ω (3) 5Ω (4) 22Ω

Ans. **(4)**

Sol. case I $i_g = \frac{E}{220+R_g} = C\theta_0$ ..(i)

Case II

$$i_g = \left(\frac{E}{220+\frac{5R_g}{5+R_g}}\right)\times\frac{5}{(R_g+5)} = \frac{C\theta_0}{5} \quad ..(ii)$$

$$\Rightarrow \frac{5E}{225R_g+1100} = \frac{C\theta_0}{5} \quad ..(ii)$$

$$\frac{E}{220+R_g} = C\theta \quad ...(i)$$

$$\Rightarrow \frac{225R_g+1100}{1100+5R_g} = 5$$

$\Rightarrow$ 5500 + 25R_g = 225R_g + 1100

200R_g = 4400

R_g = 22Ω

Ans. – 4

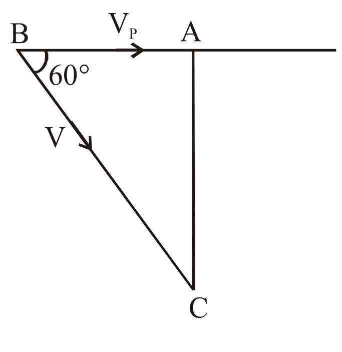
16. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is :

(1) $\frac{2\upsilon}{\sqrt{3}}$ (2) υ

(3) $\frac{\upsilon}{2}$ (4) $\frac{\sqrt{3}}{2}\upsilon$

Ans. **(3)**

Sol.



AB = $V_p \times t$

BC = Vt

$$\cos 60° = \frac{AB}{BC}$$

$$\frac{1}{2} = \frac{V_p \times t}{Vt}$$

$$V_P = \frac{V}{2}$$

17. A proton and an α-particle (with their masses in the ratio of 1:4 and charges in the ratio of 1:2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be :

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) 1 : 3 (4) $1:\sqrt{3}$

Ans. **(1)**

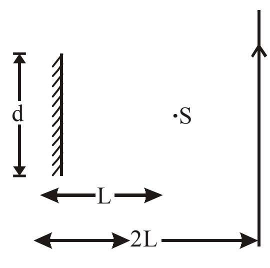
Sol. KE = qΔV

$$r = \frac{\sqrt{2mq\Delta V}}{qB}$$

$$r \propto \sqrt{\frac{m}{q}}$$

$$\frac{r_p}{r_\propto} = \frac{1}{\sqrt{2}}$$

18. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2L as shown below. The distance over which the man can see the image of the light source in the mirror is :

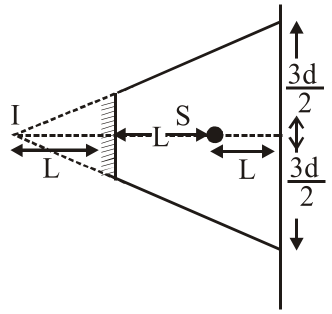


(1) 3d (2) $\frac{d}{2}$

(3) d (4) 2d

Ans. (1)

Sol.



3d

19. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5μm diameter of wire is :

(1) 50 (2) 100

(3) 200 (4) 500

Ans. (3)

Sol. Least count = $\frac{\text{Pitch}}{\text{Number of division on circular scale}}$

$$5\times10^{-6}=\frac{10^{-3}}{N}$$

N = 200

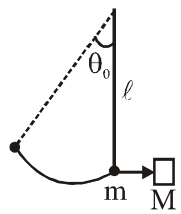
20. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by :

(1) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

(3) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (4) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

Ans. (4)

Sol.



Before colision After collision

$v=\sqrt{2g\ell(1-\cos\theta_0)}$ $v_1=\sqrt{2g\ell(1-\cos\theta_1)}$

By momentum conservation

$$m\sqrt{2g\ell(1-\cos\theta_0)}=MV_m-m\sqrt{2gl(1-\cos\theta}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right\}=MV_m$$

and $e=1=\frac{V_m+\sqrt{2g\ell(1-\cos\theta_1)}}{\sqrt{2g\ell(1-\cos\theta_0)}}$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)=V_m \quad ..(I)$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)=MV_M \quad ..(II)$$

Dividing

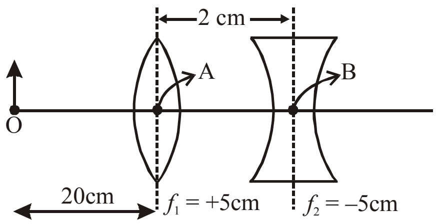
$$\frac{\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)}=\frac{M}{m}$$

By componendo divided

$$\frac{m-M}{m+M}=\frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}}=\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m}=\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\Rightarrow M=\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)m$$

21. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)



(1) 70 cm from point B at left; virtual

(2) 40 cm from point B at right; real

(3) $\frac{20}{3}$ cm from point B at right , real

(4) 70 cm from point B at right, real

Ans. **(4)**

Sol. For first lens

$$\frac{1}{V} - \frac{1}{-20} = \frac{1}{5}$$

$$V = \frac{20}{3}$$

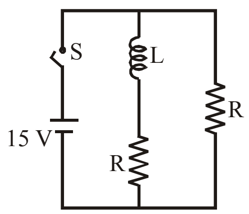
For second lens

$$V = \frac{20}{3} - 2 = \frac{14}{3}$$

$$\frac{1}{V} - \frac{1}{\frac{14}{3}} = \frac{1}{-5}$$

V = 70cm

22. In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed?

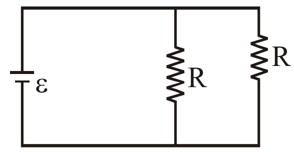


(1) 6A (2) 7.5A

(3) 5.5A (4) 3A

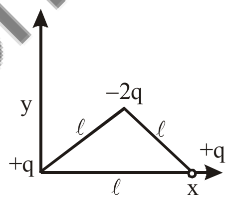
Ans. **(1)**

Sol. Ideal inductor will behave like zero resistance long time after switch is closed



$$I = \frac{2\varepsilon}{R} = \frac{2\times 15}{5} = 6A$$

23. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure:

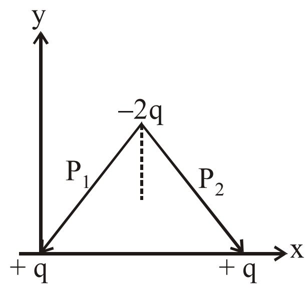


(1) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (2) $\sqrt{3}q\ell\frac{\hat{j}-\hat{i}}{\sqrt{2}}$

(3) $-\sqrt{3}q\ell\hat{j}$ (4) $2q\ell\hat{j}$

Ans. **(3)**

Sol.



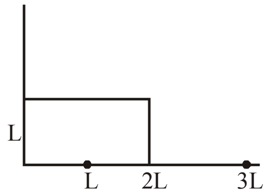
$|P_1| = q(d)$

$|P_2| = qd$

$|\text{Resultant}| = 2\,P\cos 30^\circ$

$$2\,qd\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,qd$$

24. The position vector of the centre of mass $\vec{r}\,cm$ of a symmetric uniform bar of negligible area of cross-section as shown in figure is :



(1) $\vec{r}\,cm = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

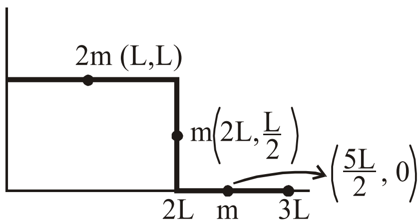
(2) $\vec{r}\,cm = \frac{11}{8}L\,\hat{x} + \frac{3}{8}L\hat{y}$

(3) $\vec{r}\,cm = \frac{3}{8}L\,\hat{x} + \frac{11}{8}L\hat{y}$

(4) $\vec{r}\,cm = \frac{5}{8}L\,\hat{x} + \frac{13}{8}L\hat{y}$

Ans. **(1)**

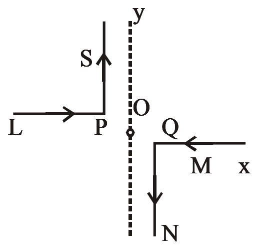
Sol.



$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

$$Y_{cm} = \frac{2m \times L + m \times \left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

25. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$) :



(1) 40 A, perpendicular into the page

(2) 40 A, perpendicular out of the page

(3) 20 A, perpendicular out of the page

(4) 20 A, perpendicular into the page

Ans. **(4)**

Sol. Magnetic field at 'O' will be done to 'PS' and 'QN' only

i.e. $B_0 = B_{PS} + B_{QN} \rightarrow$ Both inwards

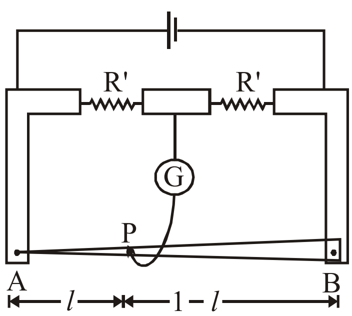
Let current in each wire = i

$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$

or $\quad 10^{-4} = \frac{\mu_0 i}{2\pi d} = \frac{2\times10^{-7}\times i}{4\times10^{-2}}$

$\therefore \quad i = 20$ A

26. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?



(1) 0.25 m (2) 0.3m

(3) 0.35 m (4) 0.2 m

Ans. **(1)**

Sol. For the given wire : $dR = C\frac{d\ell}{\sqrt{\ell}}$, where C = constant.

Let resistance of part AP is R_1 and PB is R_2

$\therefore \quad \frac{R'}{R'} = \frac{R_1}{R_2}$ or $R_1 = R_2$ By balanced WSB concept.

Now $\int dR = c\int \frac{d\ell}{\sqrt{\ell}}$

$$\therefore R_1 = C\int_0^{\ell} \ell^{-1/2} d\ell = C.2.\sqrt{\ell}$$

$$R_2 = C\int_{\ell}^{1} \ell^{-1/2} d\ell = C.(2 - 2\sqrt{\ell})$$

Putting $R_1 = R_2$

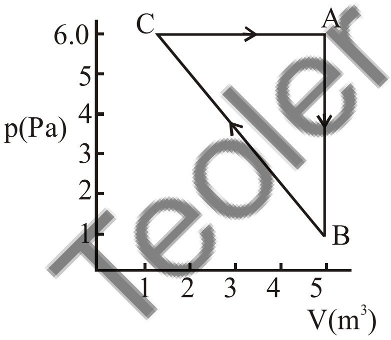
$$C2\sqrt{\ell} = C(2 - 2\sqrt{\ell})$$

$$\therefore 2\sqrt{\ell} = 1$$

$$\sqrt{\ell} = \frac{1}{2}$$

i.e. $\ell = \frac{1}{4}$ m $\Rightarrow$ 0.25 m

27. For the given cyclic process CAB as shown for a gas, the work done is :



(1) 1 J (2) 5 J
(3) 10 J (4) 30 J

Ans. (3)

Sol. Since P–V indicator diagram is given, so work done by gas is area under the cyclic diagram.

$$\therefore \Delta W = \text{Work done by gas} = \frac{1}{2} \times 4 \times 5 \text{ J}$$

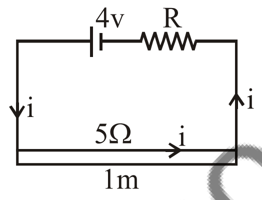
$$= 10 \text{ J}$$

28. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is :

(1) 490 Ω (2) 480 Ω
(3) 395 Ω (4) 495 Ω

Ans. (3)

Sol.



Let current flowing in the wire is i.

$$\therefore \quad i = \left(\frac{4}{R+5}\right) A$$

If resistance of 10 m length of wire is x

then $x = 0.5\ \Omega = 5 \times \frac{0.1}{1} \Omega$

$\therefore \quad \Delta V$ = P. d. on wire = i. x

$$5 \times 10^{-3} = \left(\frac{4}{R+5}\right) \cdot (0.5)$$

$$\therefore \quad \frac{4}{R+5} = 10^{-2} \quad \text{or} \quad R + 5 = 400\ \Omega$$

$$\therefore R = 395\ \Omega$$

29. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass '4 m' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to :

(1) 10.00 (2) 14.14 (3) 4.47 (4) 0.07

Ans. (2)

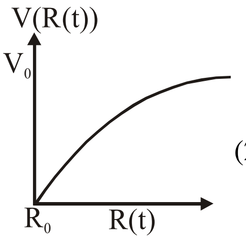
Sol. K.E. acquired by charge = K = qV

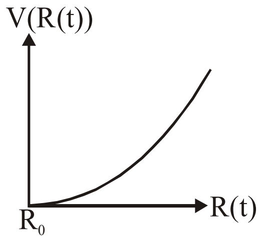
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

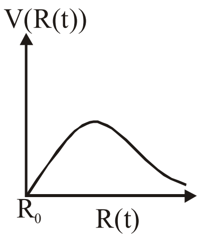
$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2m_B q_B V_B}}{\sqrt{2m_A q_A V_A}} = \sqrt{\frac{4m.q.2500}{m.q.50}} = 2\sqrt{50}$$

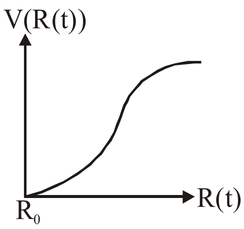
$$= 2 \times 7.07 = 14.14$$

30. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V(R(t)) of the distribution as a function of its instantaneous radius R (t) is :

(1)


(2)


(3)


(4)


Ans. (1)

Sol. At any instant 't'

Total energy of charge distribution is constant

i.e. $\frac{1}{2}mV^2 + \frac{KQ^2}{2R} = 0 + \frac{KQ^2}{2R_0}$

$\therefore \quad \frac{1}{2}mV^2 = \frac{KQ^2}{2R_0} - \frac{KQ^2}{2R}$

$\therefore \quad V = \sqrt{\frac{2}{m}\frac{KQ^2}{2}.\left(\frac{1}{R_0} - \frac{1}{R}\right)}$

$\therefore \quad V = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)} = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$

Also the slope of v-s curve will go on decreasing

$\therefore$ Graph is correctly shown by option(1)

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM

CHEMISTRY

1. Water samples with BOD values of 4 ppm and 18 ppm, respectively, are
   (1) Clean and Highly polluted
   (2) Clean and Clean
   (3) Highly polluted and Clean
   (4) Highly polluted and Highly polluted

Answer (1)

Sol. Clean water have BOD value of less than 5 ppm whereas highly polluted water could have BOD value of 17 ppm or more.

2. Given

Gas	H_2	CH_4	CO_2	SO_2
Critical Temperature/K	33	190	304	630

On the basis of data given above, predict which of the following gases shows least adsorption on a definite amount of charcoal?

(1) SO_2 (2) CO_2
(3) CH_4 (4) H_2

Answer (4)

Sol. More easily liquefiable a gas is (i.e. having higher critical temperature), the more readily it will be adsorbed.

$\therefore$ Least adsorption is shown by H_2 (least critical temperature)

3. The metal *d*-orbitals that are directly facing the ligands in $K_3[Co(CN)_6]$ are
   (1) d_{xy}, d_{xz} and d_{yz}
   (2) d_{xz}, d_{yz} and d_{z^2}
   (3) $d_{x^2-y^2}$ and d_{z^2}
   (4) d_{xy} and $d_{x^2-y^2}$

Answer (3)

Sol. $K_3[Co(CN)_6]$

During splitting in octahederal co-ordination entities, $d_{x^2-y^2}$ and d_{z^2} orbitals point towards the direction of ligands (i.e. they experience more repulsion and their energy is raised)

4. A metal on combustion in excess air forms X. X upon hydrolysis with water yields H_2O_2 and O_2 along with another product. The metal is

(1) Rb (2) Li
(3) Mg (4) Na

Answer (1)

Sol. $Rb + O_2 \xrightarrow[\text{excess}]{} RbO_2$

$2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2$

5. The correct order for acid strength of compounds $CH \equiv CH$, $CH_3 - C \equiv CH$ and $CH_2 = CH_2$ is as follows :
   (1) $CH_3 - C \equiv CH > CH \equiv CH > CH_2 = CH_2$
   (2) $CH_3 - C \equiv CH > CH_2 = CH_2 > HC \equiv CH$
   (3) $CH \equiv CH > CH_2 = CH_2 > CH_3 - C \equiv CH$
   (4) $HC \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$

Answer (4)

Sol. Order of acidic strength is

$CH \equiv CH$	>	$CH_3 - C \equiv CH$	>	$CH_2 = CH_2$
sp hybridised carbon (more electronegative)		*sp* hybridised carbon and + I effect of $—CH_3$		*sp*^{2} hybridised carbon (less electronegative)

6. The hardness of a water sample (in terms of equivalents of $CaCO_3$) containing 10^{-3} M $CaSO_4$ is

(molar mass of $CaSO_4$ = 136 g mol^{-1})

(1) 10 ppm (2) 100 ppm
(3) 90 ppm (4) 50 ppm

Answer (2)

Sol. 10^{-3} M $CaSO_4 \cong 10^{-3}$ M $CaCO_3$

$\Rightarrow$ 10^{-3} M $CaCO_3$ means 10^{-3} moles of $CaCO_3$ are present in 1L

ie 100 mg of $CaCO_3$ is present in 1L solution. Hardness of water = Number of milligram of $CaCO_3$ per litre of water.

$\therefore$ Hardness of water = 100 ppm

7. In the following reaction

Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

Aldehyde	Alcohol
HCHO	^{t}BuOH
CH_3CHO	MeOH

The best combination is

(1) HCHO and MeOH (2) HCHO and ^{t}BuOH

(3) CH_3CHO and ^{t}BuOH (4) CH_3CHO and MeOH

Answer (1)

Sol.

Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

↑ more reactive aldehyde ↑ less steric alcohol

∴ Best combination is HCHO and MeOH

8. Poly-β-hydroxybutyrate-co-β-hydroxyvalerate (PHBV) is a copolymer of ___.

(1) 3-hydroxybutanoic acid and 4-hydroxypentanoic acid

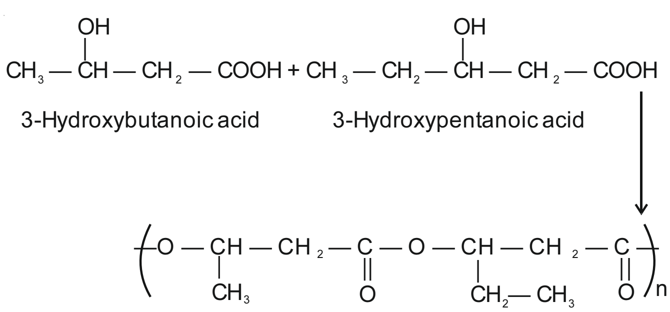
(2) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid

(3) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid

(4) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid

Answer (4)

Sol.



PHBV

∴ Monomers of PHBV are 3-Hydroxybutanoic acid and 3-Hydroxypentanoic acid.

9. The molecule that has minimum/no role in the formation of photochemical smog, is

(1) NO (2) $CH_2 = 0$

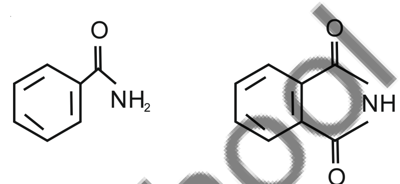
(3) O_3 (4) N_2

Answer (4)

Sol. NO, O_3 and HCHO are involved in the formation photochemical smog.

N_2 has no role in photochemical smog

10. The increasing order of reactivity of the following compounds towards reaction with alkyl halides directly is



(A) (B)

(C) (D)

(1) (A) < (B) < (C) < (D)

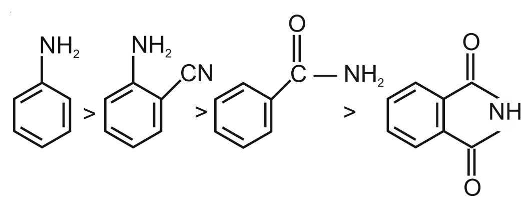
(2) (B) < (A) < (C) < (D)

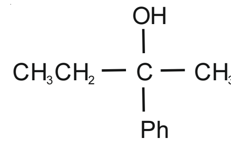
(3) (B) < (A) < (D) < (C)

(4) (A) < (C) < (D) < (B)

Answer (2)

Sol. Reactivity of compounds (nucleophiles) with alkyl halides will depend upon the availability of lone pair of electrons on nitrogen (amines or acid amides)



11.  cannot be prepared by

(1) $PhCOCH_2CH_3$ + CH_3MgX

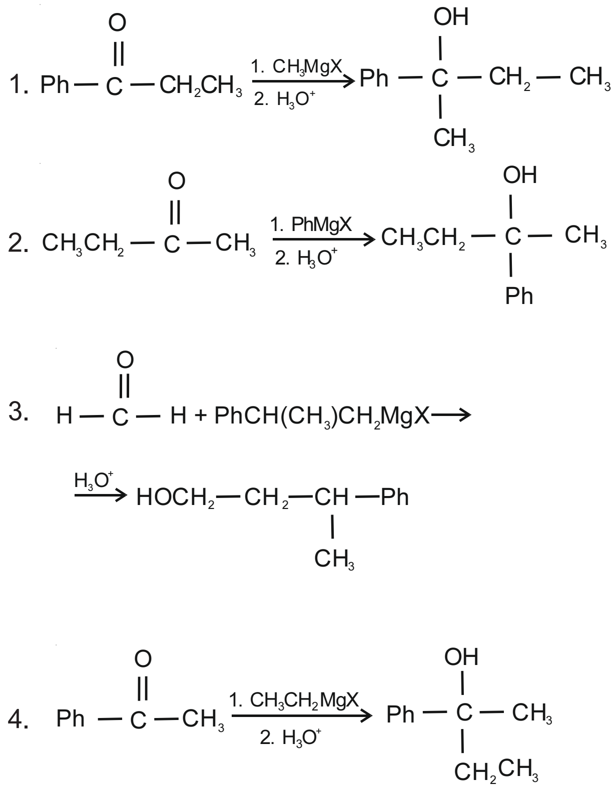
(2) $CH_3CH_2COCH_3$+ PhMgX

(3) HCHO+$PhCH(CH_3)CH_2MgX$

(4) $PhCOCH_3$+ CH_3CH_2MgX

Answer (3)

Sol.



Reaction (3) gives primary alcohol which is different from tertiary alcohol given by the remaining reactions.

12. Two solids dissociate as follows

$$A(s) \rightleftharpoons B(g) + C(g); K_{P_1} = x\ atm^2$$

$$D(s) \rightleftharpoons C(g) + E(g); K_{P_2} = y\ atm^2$$

The total pressure when both the solids dissociate simultaneously is

(1) $x^2 + y^2$ atm (2) $(x + y)$ atm

(3) $\sqrt{x + y}$ atm (4) $2\left(\sqrt{x + y}\right)$ atm

Answer (4)

Sol. $A(S) \longrightarrow \underset{P_1}{B(g)} + \underset{P_1+P_2}{C(g)}$ $K_{P_1} = P_1(P_1 + P_2) = x$

$D(S) \longrightarrow \underset{P_2+P_1}{C(g)} + \underset{P_2}{E(g)}$ $K_{P_2} = P_1(P_1 + P_2) = y$

$\therefore$ $P_1(P_1 + P_2) + P_2(P_1 + P_2) = x + y$

$\Rightarrow$ $(P_1 + P_2)^2 = x + y$

$\Rightarrow$ $P_1 + P_2 = \sqrt{x+y}$

$\therefore$ Total pressure $= 2(P_1 + P_2) = 2\left(\sqrt{x+y}\right)$ atm at equilibrium

13. The standard electrode potential $E^\ominus$ and its temperature coefficient $\left(\frac{dE^\ominus}{dT}\right)$ for a cell are 2 V and $-5 \times 10^{-4}\ VK^{-1}$ at 300 K respectively. The cell reaction is

$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$

The standard reaction enthalpy ($\Delta_r H^\ominus$) at 300 K in kJ mol^{-1} is,

[Use R = 8 JK^{-1} mol^{-1} and F = 96,000 C mol^{-1}]

(1) 206.4

(2) –384.0

(3) –412.8

(4) 192.0

Answer (3)

Sol. $\Delta_r H^\circ = -nFE^\circ + nFT\frac{dE^\circ}{dT}$

Cell reaction : $Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$

$\therefore$ $\Delta_r H^\circ = -2 \times 96000\ (2 + 300 \times 5 \times 10^{-4})$

$= -2 \times 96000\ (2 + 0.15)$

$= -412.8 \times 10^3$ J/mol

$\Delta_r H^\circ = -412.8$ kJ/mol

14. Decomposition of X exhibits a rate constant of 0.05 μg/year. How many years are required for the decomposition of 5 μg of X into 2.5 μg?

(1) 40

(2) 20

(3) 50

(4) 25

Answer (3)

Sol. Rate constant of decomposition of X = 0.05 μg/year
From unit of rate constant, it is clear that the decomposition follows zero order kinetics.

For zero order kinetics,

$[X] = [X]_0 - kt$

$\Rightarrow$ $t = \frac{5 - 2.5}{0.05}$

$= \frac{2.5}{0.05} = 50$ years

15. In the Hall-Heroult process, aluminium is formed at the cathode. The cathode is made out of

(1) Carbon

(2) Copper

(3) Platinum

(4) Pure aluminium

Answer (1)

Sol. In Hall-Heroult process, steel vessel with carbon lining acts as cathode.

16. What is the work function of the metal if the light of wavelength 4000 Å generates photoelectrons of velocity 6×10^5 ms^{-1} from it?

(Mass of electron = 9×10^{-31} kg

Velocity of light = 3×10^8 ms^{-1}

Planck's constant = 6.626×10^{-34} Js

Charge of electron = 1.6×10^{-19} JeV^{-1})

(1) 4.0 eV (2) 2.1 eV

(3) 3.1 eV (4) 0.9 eV

Answer (2)

Sol. $E_{photon} = \frac{12400}{4000} = 3.1\,eV$

$$KE_{e^-} = \frac{1}{2}mv^2 = \frac{1}{2} \times 9 \times 10^{-31} \times 36 \times 10^{10}\,J$$

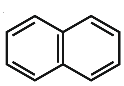
$$= 1.62 \times 10^{-19}\,J$$

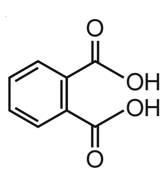
$$\simeq 1\,eV$$

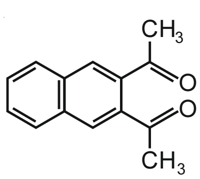
$\therefore$ Work function = 3.1 – 1

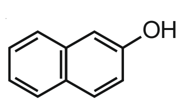
= 2.1 eV

17. Among the following four aromatic compounds, which one will have the lowest melting point?

(1) 

(2) 

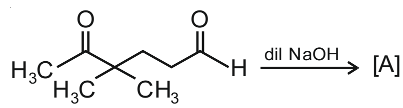
(3) 

(4) 

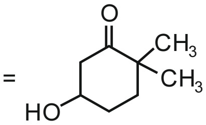
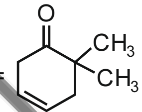
Answer (1)

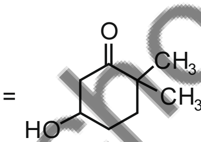
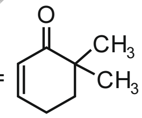
Sol. In general, polarity increases the intermolecular force of attraction and as a result increases the melting point.

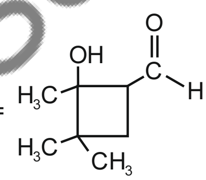
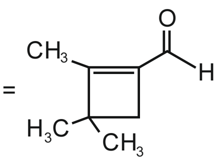
18. In the following reactions, products A and B are

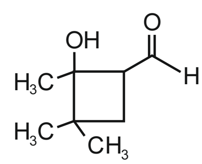
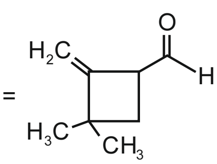


$[A] \xrightarrow[\Delta]{H_3O^+} [B]$

(1) A =  ; B = 

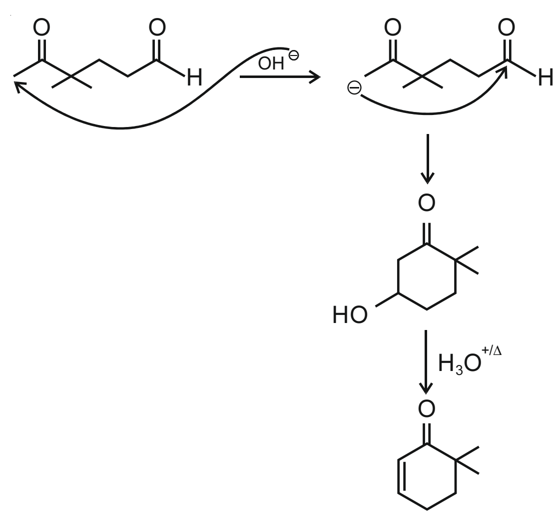
(2) A =  ; B = 

(3) A =  ; B = 

(4) A =  ; B = 

Answer (2)

Sol.



19. The pair of metal ions that can give a spin only magnetic moment of 3.9 BM for the complex $[M(H_2O)_6]Cl_2$, is

(1) V^{2+} and Co^{2+} (2) Co^{2+} and Fe^{2+}

(3) V^{2+} and Fe^{2+} (4) Cr^{2+} and Mn^{2+}

Answer (1)

Sol. μ = 3.9 BM

So, the central metal ion has 3 unpaired electrons.

∴ Configuration is either d^3 or d^7 as H_2O is a weak field ligand.

V^{2+} has d^3 configuration.

Co^{2+} has d^7 configuration.

20. In a chemical reaction, $A + 2B \overset{K}{\rightleftharpoons} 2C + D$, the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant (K) for the aforesaid chemical reaction is

(1) 1 (2) 16

(3) 4 (4) $\frac{1}{4}$

Answer (3)

Sol.

	A	+ 2B	⇌	2C	+ D
t = 0	2	3			
t_{eq}	2 – x	3 – 2x		2x	x

Given, 3 – 2x = 2 – x

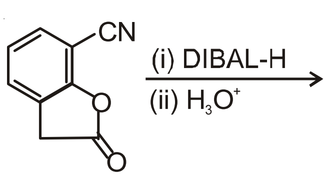
⇒ x = 1

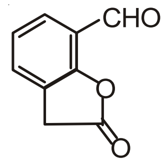
∴ [C] = 2, [D] = 1

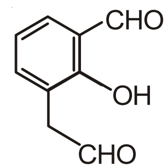
[A] = 1, [B] = 1

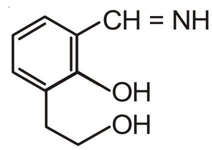
$$\therefore\ K_c = \frac{2^2 \cdot 1}{1^2 \cdot 1} = 4$$

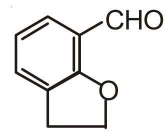
21. The major product of the following reaction



(1) 

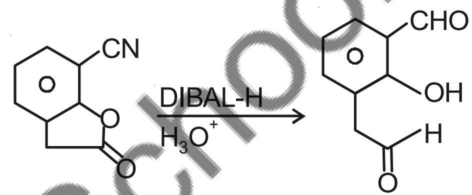
(2) 

(3) 

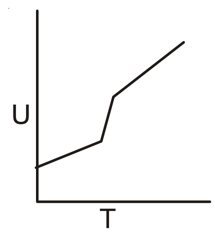
(4) 

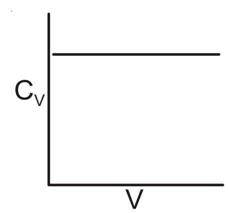
Answer (2)

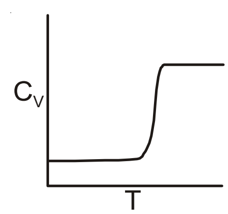
Sol. DIBAL-H followed by hydrolysis converts nitrile to aldehyde and ester to aldehyde and alcohol.

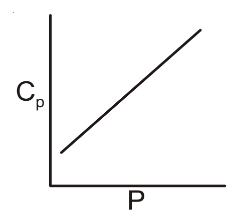


22. For a diatomic ideal gas in a closed system, which of the following plots does not correctly describe the relation between various thermodynamic quantities?

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. C_p and C_v for ideal gases are dependant on temperature only. So, C_p will not change with pressure.

23. The volume of gas A is twice than that of gas B. The compressibility factor of gas A is thrice than that of gas B at same temperature. The pressure of the gases for equal number of moles are

(1) $P_A = 2P_B$ (2) $P_A = 3P_B$

(3) $3P_A = 2P_B$ (4) $2P_A = 3P_B$

Answer (4)

Sol. $Z = \frac{PV_m}{RT}$

$\therefore \quad \frac{Z_A}{Z_B} = \frac{P_A V_A}{P_B V_B}$

$3 = \frac{P_A}{P_B} \times 2$

$2P_A = 3P_B$

24. Among the following compounds most basic amino acid is

(1) Serine

(2) Lysine

(3) Histidine

(4) Asparagine

Answer (2)

Sol. Lysine is the most basic among the given amino acids.

25. $Mn_2(CO)_{10}$ is an organometallic compound due to the presence of

(1) Mn – C bond

(2) Mn – Mn bond

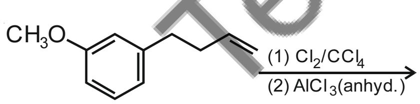
(3) Mn – O bond

(4) C – O bond

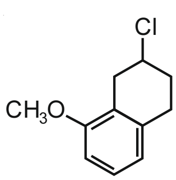
Answer (1)

Sol. It is organometallic compound due to presence of Mn – C bond.

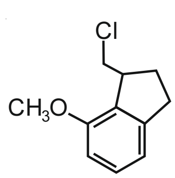
26. The major product of the following reaction is



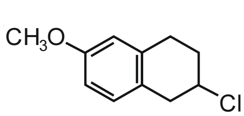
(1)



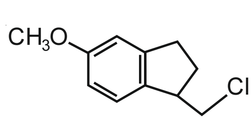
(2)



(3)

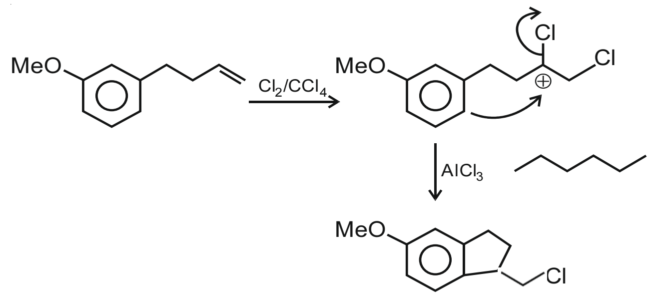


(4)



Answer (4)

Sol.



27. Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is

(1) 7 (2) 1

(3) 5 (4) 3

Answer (3)

Sol. Conc. HNO_3 oxidises I_2 to iodic acid (HIO_3).

28. The element with Z = 120 (not yet discovered) will be an/a

(1) Inner-transition metal (2) Transition metal

(3) Alkaline earth metal (4) Alkali metal

Answer (3)

Sol. Element with Z = 120 will belong to alkaline earth metals.

29. Freezing point of a 4% aqueous solution of X is equal to freezing point of 12% aqueous solution of Y. If molecular weight of X is A, then molecular weight of Y is

(1) 2A

(2) 3A

(3) A

(4) 4A

Answer (2)

Sol. $\frac{4}{M_x} = \frac{12}{M_y}$

$\Rightarrow \; M_y = 3M_x$

$\therefore \; M_y = 3A$

(Since density of solutions are not given therefore assuming molality to be equal to molarity and given % as % W/V)

30. 50 mL of 0.5 M oxalic acid is needed to neutralize 25 mL of sodium hydroxide solution. The amount of NaOH in 50 mL of the given sodium hydroxide solution is

(1) 10 g
(2) 40 g
(3) 20 g
(4) 80 g

Answer (*)

Sol. $2 \times 50 \times 0.5 = 25 \times M$

$\Rightarrow$ M = 2

$\therefore$ Moles of NaOH in 50 mL $= \frac{2 \times 50}{1000}$

$= \frac{2}{20} = \frac{1}{10}$

$\therefore$ Weight = 4 grams

No option is correct

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 09 : 30 AM To 12 : 30 PM

MATHEMATICS

1. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1+\log_e 2x\right)^2 \frac{dy}{dx}$ is equal to :

(1) $\log_e 2x$

(2) $\frac{x\log_e 2x + \log_e 2}{x}$

(3) $x\log_e 2x$

(4) $\frac{x\log_e 2x - \log_e 2}{x}$

Ans. **(4)**

Sol. $(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y = \frac{x+\ell n2}{1+\ell n2x}$$

$$y' = \frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2 = \left[\frac{x\ell n2x-\ell n2}{x}\right]$$

2. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planer, is :

(1) 2 (2) 0 (3) –1 (4) 1

Ans. **(3)**

Sol. $\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$

$\mu(\mu^2 - 1) - 1(\mu - 1) + 1(1-\mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\ \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

$\mu^3 - 1 - 3(\mu - 1) = 0$

$\mu = 1,\ \mu^2 + \mu - 2 = 0$

$\mu = 1,\ \mu = -2$

sum of distinct solutions = –1

3. Let S be the set of all points in $(-\pi,\pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

(1) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

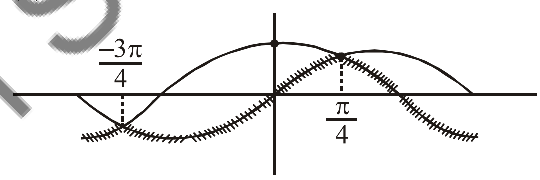
(2) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$

Ans. **(1)**

Sol.



4. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is

(1) 36 (2) 24

(3) 32 (4) 28

Ans. **(4)**

Sol. Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$\therefore\ a^3 = 512 \Rightarrow a = 8$

$\frac{8}{r} + 4, 12, 8r \rightarrow$ A.P.

$24 = \frac{8}{r} + 4 + 8r$

$r = 2,\ r = \frac{1}{2}$

$r = 2\ (4,\ 8,\ 16)$

$r = \frac{1}{2}\ (16, 8, 4)$

Sum = 28

5. The integral $\int \cos(\log_e x)dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(2) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(3) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(4) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

Ans. (2)

Sol. $I = \int \cos(\ell n x)dx$

$I = \cos(\ln x).x + \int \sin(\ell n x)\, dx$

$\cos(\ell n x)x + [\sin(\ell n x).x - \int \cos(\ell n x)dx]$

$I = \frac{x}{2}[\sin(\ell n x) + \cos(\ell n x)] + C$

6. Let $S_k = \frac{1+2+3+....+k}{k}$. If

$S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to :

(1) 303 (2) 283 (3) 156 (4) 301

Ans. (1)

Sol. $S_K = \frac{K+1}{2}$

$\Sigma S_k^2 = \frac{5}{12}A$

$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2 = \frac{2^2+3^2+--+11^2}{4} = \frac{5}{12}A$

$\frac{11\times12\times23}{6} - 1 = \frac{5}{3}A$

$505 = \frac{5}{3}A$, A = 303

7. Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :-

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$

(3) $2^{50}-1$ (4) $2^{50}+1$

Ans. (1)

Sol. S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd

$= 2^{100}-1-1[(2^{50}-1)]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50}-1)$

8. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observation is :

(1) 50 (2) 51 (3) 30 (4) 31

Ans. (4)

Sol. $\sum_{i=1}^{50}(x_i - 30) = 50$

$\Sigma x_i = 50 \times 30 = 50$

$\Sigma x_i = 50 + 50 + 30$

Mean = $\bar{x} = \frac{\Sigma x_i}{n} = \frac{50\times30+50}{50} = 30 + 1 = 31$

9. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2+y^2-18x-2y+78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :-

(1) [12, 21] (2) (2, 17)

(3) (23, 31) (4) [13, 23]

Ans. (1)

Sol. Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$

distance from S_1

$\left|\frac{3+4-\lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12, \infty]$

distance from S_2

$$\left|\frac{27+4-\lambda}{5}\right| \ge 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

so $\lambda \in [12, 21]$

10. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is :

(1) $1 : 4(16)^{\frac{1}{3}}$ (2) $1 : 2(6)^{\frac{1}{3}}$

(3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

Ans. (4)

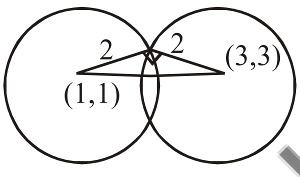
Sol. $$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4 (2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

11. let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2 = 0$ and $x^2+y^2-6x-6y+14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6 (3) 9 (4) 4

Ans. (4)

Sol.



Area = 2 × $\frac{1}{2}.4 = 2$

12. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

Ans. (2)

Sol. $\_\ \_\ \_\ \underline{4}\ \underline{4}$

$$\frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1.5^2}{6^3}\right) = \frac{175}{6^5}$$

13. If the straight line, $2x-3y+17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :-

(1) –5 (2) $-\frac{35}{3}$

(3) $\frac{35}{3}$ (4) 5

Ans. (4)

Sol. $\frac{17-\beta}{-8} \times \frac{2}{3} = -1$

$\beta = 5$

14. Let f and g be continuous functions on [0, a] such that $f(x) = f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)dx$ is equal to :-

(1) $4\int_0^a f(x)dx$ (2) $2\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $\int_0^a f(x)dx$

Ans. (2)

Sol. $I = \int_0^a f(x)g(x)dx$

$I = \int_0^a f(a-x)g(a-x)dx$

$I = \int_0^a f(x)(4-g(x)dx$

$I = 4\int_0^a f(x)dx - I$

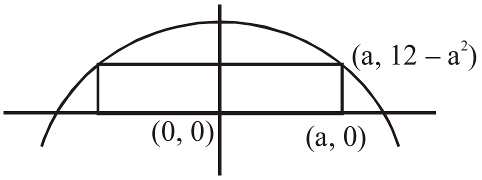
$\Rightarrow \quad I = 2\int_0^a f(x)dx$

15. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12-x^2$ such that the rectangle lies inside the parabola, is :-

(1) $20\sqrt{2}$ (2) $18\sqrt{3}$ (3) 32 (4) 36

Ans. (3)

Sol. $f(a) = 2a(12 - a)^2$



$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(2) = 32

16. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to:

(1) $p \wedge (\sim q)$ (2) $p \vee (\sim q)$

(3) $(\sim p) \wedge (\sim q)$ (4) $p \wedge q$

Ans. (3)

17. $\lim\limits_{x \to \pi/4} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is :

(1) 4 (2) $8\sqrt{2}$ (3) 8 (4) $4\sqrt{2}$

Ans. (3)

Sol. $\lim\limits_{x \to \pi/4} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$$\lim_{x \to \pi/4} \frac{(1 - \tan^4 x)}{\cos(x + \pi/4)}$$

$$2 \lim_{x \to \pi/4} \frac{(1 - \tan^2 x)}{\cos(x + \pi/4)}$$

$$R \lim_{x \to \pi/4} \frac{\cos^2 x - \sin^2 x}{\frac{\cos x - \sin x}{\sqrt{2}}} \frac{1}{\cos^2 x}$$

$$4\sqrt{2} \lim_{x \to \pi/4} (\cos x + \sin x) = 8$$

18. Considering only the principal values of inverse functions, the set $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

(1) is an empty set

(2) Contains more than two elements

(3) Contains two elements

(4) is a singleton

Ans. (4)

Sol. $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$$\Rightarrow \frac{5x}{1 - 6x^2} = 1$$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x = \frac{1}{6}$

$x = \frac{1}{6} \quad \because x > 0$

19. An ordered pair(α, β) for which the system of linear equations

$(1+\alpha)x + \beta y + z = 2$

$\alpha x + (1+\beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$ has a unique solution is

(1) (1,–3) (2) (–3,1)

(3) (2, 4) (4) (–4, 2)

Ans. (3)

Sol. For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

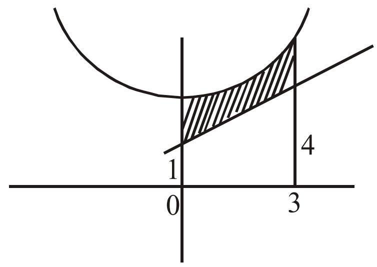
$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

20. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

(1) $\frac{15}{4}$ (2) $\frac{15}{2}$ (3) $\frac{21}{2}$ (4) $\frac{17}{4}$

Ans. (2)

Sol.



Req. area = $\int_0^3 (x^2 + 2)dx - \frac{1}{2}.5.3 = 9 + 6 - \frac{15}{2} = \frac{15}{2}$

21. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is :

(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

Ans. (2)

Sol. $3m^2x^2 + m(m-4)x + 2 = 0$

$\lambda+\frac{1}{\lambda}=1$, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1$, $\alpha^2+\beta^2=\alpha\beta$

$(\alpha+\beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2=\frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2}=\frac{6}{3m}$

$(m-4)^2 = 18$, $m = 4\pm\sqrt{18},\ 4\pm 3\sqrt{2}$

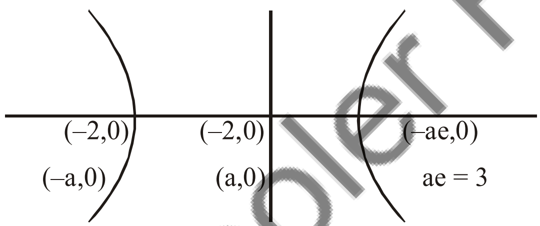
22. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?

(1) $\left(4,\sqrt{15}\right)$ (2) $\left(-6,2\sqrt{10}\right)$

(3) $\left(6,5\sqrt{2}\right)$ (4) $\left(2\sqrt{6},5\right)$

Ans. (3)

Sol.



$ae = 3$, $e=\frac{3}{2}$, $b^2=4\left(\frac{9}{4}-1\right)$, $b^2 = 5$

$\frac{x^2}{4}-\frac{y^2}{5}=1$

23. If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is :

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$

$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$

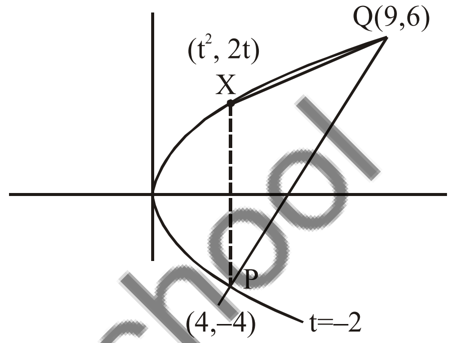
$|z|^2 = \alpha^2$, $a = \pm 2$

24. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2= 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$ (3) $\frac{625}{4}$ (4) $\frac{75}{2}$

Ans. (1)

Sol.



$y^2 = 4x$

$2yy' = 4$

$y'=\frac{1}{t}=2$, $t=\frac{1}{2}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix}\frac{1}{4} & 1 & 1\\ 9 & 6 & 1\\ 4 & -4 & 1\end{vmatrix}=\frac{125}{4}$$

25. the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

(1) $11/\sqrt{6}$ (2) $6\sqrt{11}$ (3) 11 (4) $11\sqrt{6}$

Ans. (1)

Sol. $\begin{vmatrix}i & j & k\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}$

$\hat{i}(35-28)-\hat{j}(21.7)+\hat{k}(12-5)$

$7\hat{i}-14\hat{j}+7\hat{k}$

$\hat{i}-2\hat{j}+\hat{k}$

$1(x+2)-2(y-2)+1(z+15)=0$

$x-2y+z+11=0$

$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

26. The maximum value of $3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{19}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{34}$

Ans. (1)

Sol. $y=3\cos\theta+5\left(\sin\theta\frac{\sqrt{3}}{2}-\cos\theta\frac{1}{2}\right)$

$\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$

$y_{max}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

27. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

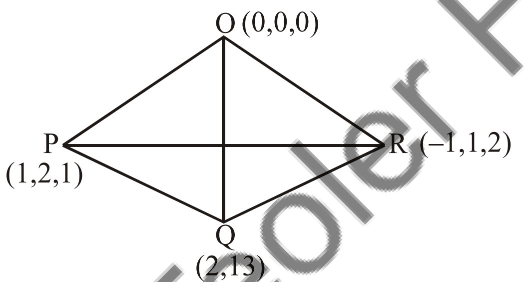
(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

Ans. (2)

Sol. $\overrightarrow{OP}\times\overrightarrow{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$5\hat{i}-\hat{j}-3\hat{k}$



$\overrightarrow{PQ}\times\overrightarrow{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$\hat{i}-5\hat{j}-3\hat{k}$

$\cos\theta=\frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2}=\frac{19}{35}$

28. Lety = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x,(x>1)$. If $2y(2)=\log_e 4-1$, then y(e) is equal to :-

(1) $\frac{e^2}{4}$ (2) $\frac{e}{4}$ (3) $-\frac{e}{2}$ (4) $-\frac{e^2}{2}$

Ans. (2)

Sol. $\frac{dy}{dx}=\frac{y}{x}=\ell n\,x$

$e^{\int\frac{1}{x}dx}=x$

$xy=\int x\ell nx+C$

$\ell n\,x\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}$

$xy=\frac{x}{2}\ell n\,x-\frac{x^2}{4}$ +C, for 2y(2)= 2ℓn2 – 1

⇒ C = 0

$y=\frac{x}{2}\ell n\,x-\frac{x}{4}$

$y(e)=\frac{e}{4}$

29. Let $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$ and $Q=[q_{ij}]$ be two 3×3 matrices such that $Q-P^5=I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to:

(1) 15 (2) 9 (3) 135 (4) 10

Ans. (4)

Sol. $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$

$P^2=\begin{bmatrix}1&0&0\\3+3&1&0\\9+9+9&3+3&1\end{bmatrix}$

$P^3=\begin{bmatrix}1&0&0\\3+3+3&1&0\\6.9&3+3+3&1\end{bmatrix}$

$P^n=\begin{bmatrix}1&0&0\\3n&1&0\\\frac{n(n+1)}{2}3^2&3n&1\end{bmatrix}$

$P^5=\begin{bmatrix}1&0&0\\5.3&1&0\\15.9&5.3&1\end{bmatrix}$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

30. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 82 (2) 240 (3) 164 (4) 120

Ans. (4)

Sol. No. of ways = ${}^{10}C_3$ = 120